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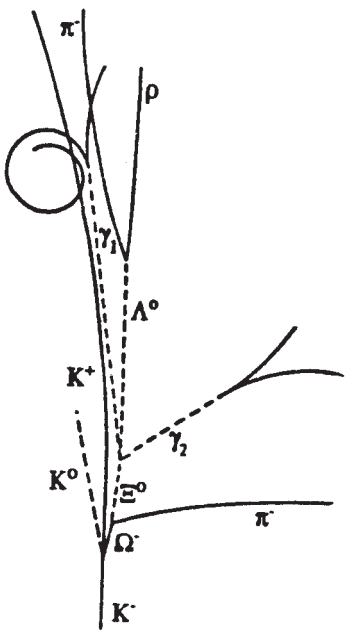

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NONUNITARY LIE-ISOTOPIC AND LIE-ADMISSIBLE SCATTERING THEORIES OF HADRONIC MECHANICS, I: Conceptual and Mathematical Foundations, 447

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NONUNITARY LIE-ISOTOPIC AND LIE-ADMISSIBLE SCATTERING THEORIES OF HADRONIC MECHANICS, III: Basic Lie-Isotopic Formulation without Divergencies, 571

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NONUNITARY LIE-ISOTOPIC AND LIE-ADMISSIBLE SCATTERING THEORIES OF HADRONIC MECHANICS, I: Conceptual and Mathematical Foundations

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Submission History

This paper and the remaining four of this series have been rejected by: G.t Hooft, editor of *Foundations of Physics;* L. Cifarelli, editor of *La Rivista Nuovo Cimento;* B. Nachtergaele, editor of *Journal of Mathematical Physics;* and E. Paal, editor of the *Journal of Generalized Lie Theories and Applications.* Additionally, Santilli stated *"I have not submitted the papers to APS and IoP journals due to their rejection of 'all' papers in the field since my last publications there in 1980* [PR D **22**, 892, 1980] *and 1983* [JoPG **18**, L141, 1983] *respectictively, all papers published in other refereed journals around the world.* In particular, t Hooft rejection was via a form letter without technical review; Cifarelli rejection included scientific wanderings plus the statement *"Moreover we do not wish receiving any further extension of your work.";* Nachtergaele blamed all JMP editors with the statement *"There was no support from the Editorial Board to publish the manuscripts."* (see the web site http://www.scientificethics.org/JMP.htm); and Paal's obstructed the very submission. The publisher submitted the papers to three outstanding physicists who released the following reviews: 1) *Following a lifelong research, Prof. Santilli has identified vast evidence on non-unitary effects in high energy inelastic scatterings requiring a re-inspection of 'experimental results' claimed via conventional unitary elaborations, with scientific, financial and ethical implications such that the suppression of the papers without due scientific process would constitute a crime against society;* 2) *Prof. Santilli's penetrating and impecccable analysis of scattering events establishes the need for the re-examination of basic paradigms due to experimentally established non-Lagrangian and non-Hamiltonian effects within the scattering region that must be debated in refereed publications for our community to verify minimal standards of ethic s and accountability;* 3) *Prof. Santilli's papers are: professionally written, technically outstanding, and potentially historical.* Therefore, without any consultation of the editors of the *Hadronic Journal,* the publisher elected to print the papers with the request to t Hooft, Cifarelli, Nachtergaele, Paal, APS-IoP editors and their friends that, in order to dispel increasing allegations of acting under orders by a scientific ring, they should publish their technical objections, if any possible, because science solely advances via publications and certainly not via equivocal 'reviews' voiced from opaque academic corridors. The series of five papers was finally accepted for publication by Kathmandu University, Nepal, following international review by qualified ethical scientists. The reasons for the political handling of these submissions is explained and denounced in the post by J. V. Kadeisvili Obscurantism at CERN on God's Particle? http://www.santilli-foundation.org/cern-obscurantism.php

This Submission History has been released by R. M. Santilli in his capacity as Founder and Honorary Editor in Chief of the Hadronic Journal without any consultation with any of the Editors.

Abstract

A nonunitary extension of the conventional, unitary scattering theory has been advocated by various authors as an effective way to incorporate nonpotential effects expected in dissipative nuclear events, deep mutual penetrations of the wavepackets of scattering particles, and other events. Nevertheless, these efforts had to be abandoned because of the violation of causality, lack of conservation of probabilities, and other problems emerging under nonunitary time evolutions. We show that the reformulation of a nonunitary scattering theory permitted by the isotopic branch of hadronic mechanics and its underlying Lie-isotopic theory, here presented under the name of *isoscattering theory,* reconstructs unitarity on iso-Hilbert spaces over isofields, a property known as *isounitarity,* thus resolving said problematic aspects, while having no divergencies *an initio,* and providing a significant broadening of the quantum scattering theory, although the Lie-isotopic theory is expected as being solely applicable to *reversible scattering events.* This first paper is devoted to the conceptual and mathematical foundations of the Lie-isotopic scattering theory, including the resolution of the inconsistencies of nonunitary theories. The physical foundations, the absence of divergencies from primitive axioms, and initial comparisons of the elaboration of measured quantities (cross sections, scattering angles, etc.) via the Lie and the Lie-isotopic scattering theories for reversible scattering events are studied in subsequent papers. Deep inelastic events are *irreversible over time,* thus requiring the further Lie-admissible broadening of the formalism studied in subsequent papers. ..

Key words scattering theories, nonunitary theories, isounitary theories
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1. Introduction

As it is well known, the conventional, quantum mechanical, scattering theory (see, e.g., Ref. [1] and literature quoted therein), has permitted historical advances in the 20th century particle physics. Nevertheless, physics is a discipline that will never admit final theories because all theories are a mere approximation of the complexities of nature. No matter how accurate a given theory may be perceived, its broadening for a more accurate representation of nature is only a question of time.

In fact, numerous authoritative doubts on the final character of the conventional quantum scattering theory have been expressed, such as:

1) P. A. M. Dirac (see, e.g., Ref. [2]) expressed in 1981 serious concerns for the infinities in scattering theories and indicated the need for a revised theory avoiding divergences $ab\ initio,$ rather than via $ad\ hoc$ procedures of unknown physical origin;

2) B. Davies (see Ref. [3] and papers quoted therein) voiced in 1981 the need to extend the scattering theory into a nonunitary form so as to incorporate imaginary potentials as used in dissipative nuclear effects and other events;

3) W. Heisenberg (see the review in Ref. [5])voiced the need for a nonlinear extension of quantum mechanics, due to the known nonlinear character of nature;

4) Einstein, Podolsky and Rosen expressed their celebrated dout on the "lack of completion" of quantum mechanics (see later on for comments);

5) R. M. Santilli [4,5] suggested in 1978 the construction of a nonunitary covering of quantum mechanics under the name of $hadronic\ mechanics$ in order to lift the quantum assumption of point-like particles into a form admitting a representation of the actual extended, thus generally nonspherical and deformable character of the wavepackets and/or charge distributions of particles, a representation of contact non-Hamiltonian interactions expected in deep overlapping of scattering particles, and other effects beyond the representational capabilities of quantum mechanics.

The above initial efforts subsequently resulted as being afflicted, in their original formulation, by fundamental inconsistencies. In essence, a

theory along the above lines generally requires non-Hamiltonian effects (i.e., effects not representable with a Hermitean Hamiltonian), a feature causing the time evolution of the theory as being no longer unitary. In turn, the loss of unitarity implies: the loss of Hermiticity, thus observability, over time (an occurrence known as *Lopez lemma* [6]); the violation of causality; the lack of conservation of probabilities; the inability to predict the same numerical values under the same conditions at different times; and other basic problematic aspects known under the name of *Theorems of Catastrophic Inconsistencies of Nonunitary Theories* [6-11] (see also the review in Ref. [16a]).

A resolution of the above inconsistencies required the construction of a new mathematics, today known as *isomathematics,* based on the isotopic (i.e., axiom-preserving) lifting of the basic unit $\hbar = 1$ of quantum mechanics into the most general possible, positive-definite, integro-differential operator with an explicit dependence on any desired local quantity $\hat{I}(t, r, p, \psi, ...) = 1/\hat{T}(t, r, p, \psi, ...) > 0$ known as *isounit;* its inverse $\hat{T}$ being known as the *isotopic element.* The isotopic lifting of the basic (left and right) unity $\hbar \to \hat{I}(t, r, p, \psi, ...)$ then required corresponding compatible isotopies of the entire mathematics of quantum mechanics, including the isotopic lifting of fields, spaces, functional analysis, differential calculus, topology, geometries, algebras, groups, symmetries, representation theory, etc. [12-25].

While quantum systems are entirely represented by the sole knowledge of the Hamiltonian $H(r, p) = p^2/2m + V(r)$, the representation with hadronic mechanics of extended particles at short mutual distances requires the knowledge of *two* quantities, the usual Hamiltonian $H(r, p) = p^2/2m + V(r)$ for the representation of action-at-a-distance, potential interactions, plus the isounit $\hat{I}(t, r, p, \psi, ...)$ for the representation of the actual size, shape and density of particles, their contact nonpotential interactions and other features beyond any hope of representation via a Hamiltonian. Note that, being an operator by assumption, the isounit does not commute with the Hamiltonian and, therefore, it is not generally a constant (although it is at times averaged into a constant).

By remembering that the unit is the basic invariant of any theory, the representation of non-Hamiltonian features and interactions via the

isounit is the only form known to the authors permitting nonunitary theories to achieve the crucial invariance over time as possessed by the majestic axiomatic consistency of unitary quantum theories. The resolution of the remaining inconsistencies of early nonunitary theories was achieved via the reconstruction of unitarity over iso-Hilbert spaces over isofields, a property known as $isounitarity$ (see the revioew below).

Mathematical maturity was achieved with: the discovery in 1993 that the conventional axioms of numerical fields admit basically new realizations of real, complex and quaternionic numbers with arbitrary (left and right, positive-definite) units, thus resulting in basically new numbers [12]; the discovery in 1995 of the dependence by the conventional differential calculus on the assumed basic unit with the consequential emergence of new calculi [13]; the isotopies in 1998 of the fundamental $SU(2)$ spin and isospin symmetries with consequential revision of Bell's inequality and all that [14]; and other advances identified later on. The achievement of physical maturity was then consequential, and so were numerous applications and experimental verifications (see monographs [15] of 1995, updates [16] of 2008, books [17-24] and vast literature quoted therein).

In these papers, we present the reformulation of nonunitary scattering theories permitted by the isotopic branch of hadronic mechanics that is based on the Lie-$Santilli$ $isotheory$ and related $isomathematics$ [4,14-23]. Since all isounits assumed in these papers are Hermitean from their positive-definiteness, $\hat{I} = \hat{I}^\dagger > 0$, such a reformulation is primarily intended for $scattering\ processes\ that\ are\ reversible\ over\ time,$ hereon called $isoscattering\ theory,$ whose prefix "iso" is intended to indicate the preservation of the abstract axioms of the quantum scattering theory and merely present a $broader\ realization.$

Hence, the reader should be aware from these introductory lines that a main effort of these initial papers on isotopies is that of $preserving\ the\ abstract\ axioms\ of\ special\ relativity,\ quantum\ mechanics\ and\ the\ conventional\ scattering\ theory,$ and studying their broader realization permitted by the novel mathematics. The non triviality of these isotopic liftings will then be illustrated by showing that the scattering theory: 1) Resolves the inconsistencies of nonunitary theories; 2) Avoids divergences ab ini-

tial; and 3) Broadens the representational capability of the conventional scattering theory with the representation of conventional potential interactions represented by the conventional Hamiltonian H, plus nonpotential interactions represented by the isotonic $\hat{I}$ caused by the deep mutual penetration of particles as customary in high energy scattering events, and the direct geometric representation of the size, shape and density of the scattering region. The issue as to whether the numerical values characterized by the scattering theory are different than those characterized by the conventional theory for the same measured quantities, can only be addressed subsequently.

It should be stressed that *the extension of the formalism to irreversible processes requires a yet broader irreversible mathematics,* known as *Lie-admissible mathematics,* which is characterized by *two non-symmetric units,* $\hat{I}^{>},^{<} \hat{I}$ for motions forward and backward in time, respectively. In turn, such basic assumptions require a step-by-step Lie-admissible lifting of the entire isotopic formalism [4,15]. Due to its complexity, this broader formulation cannot possibly be presented in these first papers, and will be presented at some later time (see monographs [15] and the latest memoir [25].

Hence, *the reversible scattering theory presented in these papers is a mandatory intermediate step prior to the construction of the irreversible Lie-admissible scattering theory and related new mathematics* known under the names of *scattering theory* and *mathematics* where the prefix "geno" was suggested since the original proposal of 1978 [4.5] to indicate from its Greek meaning that, this time, the axioms of special relativity, quantum mechanics and the scattering theory are abandoned in view of their notorious reversible character (see next section) in favor of new, structurally broader, irreversible axioms.

It should be indicated that in these papers we present, apparently for the first time, the axiomatic foundations of the isoscattering theory, although the main elements of the new theory have been known for some time, but often ignored by physicists dealing with scattering processes to their peril. In fact, the following basic results have been available in the scientific literature for some time:

I) *Convergent perturbation theory.* Recall that quantum mechanics is

based on the well known Lie product $[A, B] = AB - BA$ between generic matrices or operators A, B, while the isotopic branch of hadronic mechanics is based on the Lie-Santilli isoproduct $[A\hat{,}B] = A\hat{T}B - B\hat{T}A$, first presented in Ref. [4] of 1978 and then studied by the authors in various works (see the review in Refs. [15,16]), where T is the inverse of the isounit. It was then easy to see since the original proposal to build hadronic mechanics [5] that any divergent (or weakly convergent) canonical series $A(w) = A(0) + w(AH - HA)/1! + ... \to \infty$ can be turned into a strongly convergent form under the lifting $A(w) = A(0) + w(A\hat{T}H - H\hat{T}A)/1! + ...$ for all isotopic element $\hat{T}$ sufficiently smaller (in absolute value) than w, a feature naturally verified by actual models, as we shall see. This feature was then studied by A. Jannussis and other authors (for brevity, see Chapter 11 of monograph [15b] for review and references).

II) *Conservation of probability.* As it is well known, the quantum S-matrix is unitary as a condition to preserve probabilities [1]. Hence, it was popularly believed that nonunitary theories violate the conservation of probabilities. The recovering of the conservation of probability under an isounitary reformulation of nonunitary theory was well established by 1995 [15b].

III) *Absence of divergencies.* Recall that divergencies in quantum scattering theories mainly originate from Dirac's delta function $\delta(x - x_0)$ since the latter is divergent at $x = x_0$ [1]. The absence of divergencies in the scattering theory of hadronic mechanics was identified in 1982 by Myung and Santilli [26] with the introduction of the isotopic covering of the Dirac delta function called by Nishioka [27] the *Dirac-Myung-Santilli isodelta function* and denoted $\hat{\delta}(x - x_0) = \delta[\hat{T}(x - x_o)]$ which, as one can see, removes the divergency of the delta function at $x = x_0$ under a judicious choice of the isotopic element T, as reviewed later on in Section 3.8.

IV) *Nonpotential scattering theory.* The extension of the quantum scattering theory to incorporate interactions not entirely represented with a Hamiltonian, as expected in deed inelastic scattering, was sufficiently voiced in the original proposal [5], and subsequently studied by R. Mignani [28] and others. Additional more recent studies on nonpotential scattering theory have been conducted by A. K. Aringazin et al [29] (again for

brevity, see Chapter 12 of monograph [15b] for reviews and additional references).

V) *Inequivalence of Hamiltonian and non-Hamiltonian data elaborations.* It is popularly believed that, since cross sections, scattering angles and other quantities are measured, the numerical values produced by data elaborations via unitary scattering theories have a final experimental character. In reality, nature is not as simple as all of us tend to believe. Santilli showed in 1989 (see the review in Chapter 12 of monograph [15b]) that the *elaboration of measured quantities via quantum and hadronic scattering theories are generally inequivalent,* thus warranting serious comparative studies. This is due to the fact known since 1978 [5] that, if the Hamiltonian H of a given scattering theory has the eigenvalue E, $H|\psi>=E|\psi>$, *the same Hamiltonian* H *has a generaly different eigenvalue* E' *for the isoscattering theory,* $HT|\psi'>=E'|\psi>$ $E'\neq E$, trivially, in view of the general lack of commutativity between H and T (see Section 3.6 for details). Irrespective of all preceding aspects, the latter occurrence, alone, warrants a reinspection of the conventional, reversible, Hamiltonian, unitary scattering theory.

The reader should be aware from these introductory lines of the existence of preliminary, yet rather vast experimental support of deviations from conventional Lie theories in virtually all quantitative sciences when dealing with the main assumption of the scattering theory, that is, extended particles and electromagnetic waves moving within physical media. Among these experimental data, we mention:

A) The need for contact non-Hamiltonian interactions to achieve an actual *attractive* force between the *identical* electrons in molecular valence couplings since, as expected to be known although rarely voiced, identical electrons repel each other according to quantum mechanics and chemistry [32];

B) Deviations from the geometry of spacetime have been, again preliminarily, yet directly measured in the experimental verification of the *isoredshift,* [31]. We are here referring to a shift toward the red of the frequency of light propagating within a transparent physical medium without any relative motion between the source, the medium and the detector, the shift being merely due to the loss of energy $E=h\nu$ by light

to the medium due to inevitable interactions, with consequential evident reduction of frequency.

C) The elaboration of numerous particle physics experiments dealing with the hyperdense interior of hadrons, when elaborated without $ad\ hoc$ parameters or arbitrary functions of unknown physical origin, show the clear presence of non-Hamiltonian effects [16d]. This is typically the case of the two-point amplitude of the Bose-Einstein correlation whose quantum fit of experimental data requires $four$ arbitrary parameters (the so-called "chaoticity parameters"), while vacuum expectation values admit at best two parameters. These effects can be fully representable via a four-dimensional isounit of which the three space components represent the actual, very elongated shape of the proton-anti proton fireball, and the forth components represents its density, in remarkable agreement with experimental data [$loc.\ cit.$].

In any case, as part of the ongoing efforts to appraise the experimental claims based on the conventional scattering theory, a rather significant experimental effort is under way at this writing (Spring 2010) to repeat within physical media the historical experiments that have established the validity of special relativity, all done in vacuum, as well known. This significant experimental effort on the disciplines actually holding within physical media at large, and within the scattering region in particular, combined with the theoretical efforts herein considered, will eventually provide the necessary elements for the resolution of fundamental open issues in scattering experiments, of course, in due time.

Above all, the reader should keep in mind that special relativity and quantum mechanics are reversible theories, thus having manifest limitation for all energy releasing processes, due to their strict irreversibility. Therefore, the conception, quantitative treatment and experimental verification of much needed new clean energies, such as the novel $Intermediate\ Controlled\ Nuclear\ Fusions$ (ICNF) [37] are crucially dependent on the covering formulations treated in these papers. Their possible confirmation in particle accelerators via the covering isoscattering theory would then acquire a primary significant for the resolution of alarming environmental problems.

For additional historical data and a comprehensive literature in the

field, interested colleagues may inspect Refs. [16], particularly the *General Bibliography of Hadronicx Mechanics* in Volume [16a].

In closing these introductory lines, we should recall that the conventional scattering theory achieved maturity only following decades of collegial studies presented in a large number of refereed publications. Consequently, it is hoped the reader is not expecting a final resolution of the scattering problem in these initial papers, but merely the *initiation* of the studies leading to a future collegial resolution following a predictable large number of additional papers.

2. Basic Physical Assumptions

2.1. Exterior and Interior Dynamical Problems. Until the earlier part of the 20th century, there was a clear distinction between (see Refs. [30] for technical characterizations):

1) *exterior dynamical problems,* referred to systems of point-particles and electromagnetic waves propagating in empty space; and

2) *interior dynamical problems,* referred to extended particles and electromagnetic waves propagating within physical media.

As a historical note, we recall that Schwartzschild wrote *two* papers, the first on the exterior gravitational problem containing his celebrated solution, and a second, virtually ignored paper on the interior gravitational problem (for review and references, see Ref. [15a]).

The primary difference between exterior and interior problems is that the former verify the integrability conditions for the existence of a Lagrangian or a Hamiltonian (the so-called *conditions of variational selfadjointness*), while the latter systems (called *variationally nonselfadjoint*) violate these conditions due to the presence of contact, nonconservative and nonpotential interactions, thus not being representable with Lagrangian or Hamiltonian mechanics [30].

With the advent of special and general relativities, the distinction between exterior and interior dynamical problems was eliminated via the reduction of interior problems to a finite number of point-particles that, as such, move in vacuum, thus recovering the conditions of exterior problems.

2.2. No Reduction Theorems. In the second half of the 20th century, it became known that interior dynamical problems cannot be consistently reduced to exterior problems, an occurrence known under the name of *No reduction Theorems,* such as:

NO REDUCTION THEOREM 2.1 [5,25]: A macroscopic system in nonconservative and irreversible conditions cannot be consistently reduced to a finite collection of point-like particles all in conservative and reversible conditions and, vice versa, the latter system cannot consistently reconstruct the former under the correspondence or other principles.

A number of additional No Reduction Theorems were also proved based on the violation of thermodynamical laws due to the evident loss of entropy when passing from a real physical system to an ideal collection of point-particles moving in empty space all in conservative conditions, as necessary to verify special relativity, quantum mechanics and the conventional scattering theory.

An additional popular belief disproved by the No Reduction Theorems is that total conservation laws for an isolated system are solely verified by a system of particles in conservative conditions. In fact, it was proved in Ref. [30b] that, since they have no potential energy, nonconservative forces are in essence exchange forces, as a result of which they cancel each other when the system is isolated, resulting in the full verification of the conventional total conservation laws.

Yet another popular belief dispelled by the above No reduction Theorems is that the nonlinear, nonlocal-integral and nonpotential forces of our macroscopic environment "disappear" in the reduction of an interior system to its elementary constituents. As a matter of fact, No Reduction Theorem 2.1 establishes that *the nonlinear, nonlocal and nonpotential forces experienced, for instance, by a spaceship during reentry in our atmosphere originate at the most primitive possible level, that of elementary particles,* and are evidently due to the interactions of the electron orbitals of the peripheral atoms constituting the spaceship with the electron orbitals of the resistive medium (see Figure 1).

The particular type of non-Hamiltonian interactions here referred to

Figure 1: *A suggestive view from NASA of a spaceship during reentry in our atmosphere. Recent No reduction Theorems have established that the nonlinear, nonlocal-integral and nonpotential non-Hamiltonian forces experienced by the spaceship originate at the ultimate elementary level of nature, thus being also present in the interior of the scattering region at high energies.*

deals with the deep overlapping of the wave packets and/or the charge distribution of particles and are referred to as *nonlocal-integral interactions* (or merely nonlocal for brevity) in the sense that they occur over a surface or volume integral. As such, the nonlocal interactions at the basis of the scattering theory cannot be reduced, by conception, to a finite set of isolated points.

Note that we are including nonlocal interactions experienced by electrons, namely, by particles with a notorious *point-like charge.* Nevertheless, electrons do not have a "point-like wavepacket," thus experiencing indeed nonlinear, nonlocal and nonpotential interactions when in conditions of deep mutual penetration, as it is the case for valence electron coupling in molecular structures [32].

The studies reported in Refs. [30] have also established that the time evolution of systems with nonlinear, nonlocal and nonpotential interaction are necessarily noncanonical at the classical level and nonunitary at the operator level. We are now minimally equipped to formulate the following:

ASSUMPTION 2.1: The scattering region is an interior dynamical system, thus characterized by a nonlinear (in the wavefunction), nonlocal (integral) and nonpotential (nonunitary) time evolution.

Note that the No Reduction Theorems prohibit the exact reduction of the scattering region to a finite set of isolated points, which is considered a mere first approximation of a rather complex reality. The same theorems identify the evident need for covering formulations. Note finally that the No Reduction Theorems are not bypassed by the reduction of the scattering particles to point-like quarks, since elementary constituents with a point-like wavepacket do not exist.

2.3. Insufficiencies of the Lorentz-Poincaré Symmetry. The breaking of the *Lorentz-Poincaré (LP) symmetry* for interior dynamical problems at large, and particularly for the interior of the scattering region, is rather plausible and should be studied seriously because no scattering theory can claim final results until the basic spacetime symmetry is established beyond scientific doubt. Among a number of symmetry breaking aspects, we quote [15,16]:

1) The axiomatic foundations of the Lorentz-Poincaré symmetry requires the equivalence of all inertial reference frames. This feature is certainly valid in empty space, but it is unresolved for the interior of the scattering region because of the impossibility of even defining inertial reference frames in *interior* conditions. Inertial reference frames are indeed used in quantum scattering theories, but they constitute an *exterior* treatment, thus reducing an interior to an exterior problem. Additionally, in vacuum there is no known experimental way to detect a privileged reference frame, as well known (Michelson-Morley experiment). By contrast, the sole reference frame that can be consistently defined for the scattering region is the privileged reference frame at rest with the interior region itself, since other frames would require motion of a hypothetical observer within a hyperdense medium.

2) The Lorentz-Poincaré symmetry is exactly valid for *Keplerian systems,* that is, systems of point-particles moving in vacuum around a heavier particle known as the *Keplerian nucleus.* By contract, *the scattering*

region has no Keplerian nucleus. This aspect alone may cause a breaking of the Lorentz-Poincaré symmetry.

3) There are serious reasons to expect that the historical experiments that have established the validity of special relativity in vacuum are invalid in interior conditions [31]. For instance, it is easy to see that., in the event the known Fizeau experiment is repeated entirely underwater, there are contributions to the travel of light in water outside the traditional pipes with opposite water velocities that violate Lorentz-transformations. By contrast, the repetition of the Michelson-Morley experiment under complete underwater conditions is expected to retain the original result, this time confirming the constancy of the speed of light with respect to the privileged reference frame at rest with the water, by therefore no longer confirming the Lorentz symmetry. Needless to say, a problem of such a fundamental character cannot be resolved in a few sentences one way or another, and requires the systematic repetition in interior conditions of all historical experiments that have established the validity of the Lorentz-Poincaré symmetry in vacuum [31].

The reader should be aware that a rather vast effort has been conducted over decades for the construction of a covering spacetime symmetry applicable to interior problems at large, and the scattering region in particular. These efforts required first the construction of the covering *Lie-Santilli isotheory* [4,15,16,18-34] capable of reducing nonlinear, nonlocal and noncanonical (or nonunitary) interior problems to equivalent *isolinear, isolocal and isocanonical (or isounitary) forms* (see next section for details). Only following the achievement of the Lie-Santilli isotheory, the efforts could be concentrated in the construction of a covering of the Lorentz-Poincaré symmetry applicable to interior conditions [33-43] which is known today as the *Lorentz-Poincaré-Santilli (LPS) isosymmetry.* We reach in this way the following:

ASSUMPTION 2.2: The scattering region is characterized by the Lorentz-Poincarś-Santilli isosymmetry.

The noninitiated reader should know that, by conception and construction, *the LPS isosymmetry is locally isomorphic to the conventional LP symmetry.* This feature may have deep implications for the scattering

problem because, in the final analysis, it may imply that the data elaboration of existing high energy experiments with the conventional and the isotopic scattering theory yields the same numerical value. Rather than being a drawback, if established by future collegial works, this possible outcome alone warrants this study, e.g., because it would established the validity of special relativity for interior conditions nowadays considered inapplicable.

In any case, as shown in paper III of this series, even assuming that the elaboration of past experiments via the conventional and isotopic scattering theory yields the same numerical results, the broader representational capabilities of the isotopic theory are beyond doubt, thus offering the possible prediction and representation of scattering events beyond the capability of the conventional theory; It is only hoped the reader does not expect the final resolutions of these complex issues in these initial papers.

2.4. Insufficiencies of Quantum Mechanics. Following the historical successes of quantum mechanics for the structure of the hydrogen atoms and numerous other systems, quantum mechanics has been applied to all possible particle conditions existing in the universe, thus including interior conditions, as typically occurring in the scattering region as well as in the structure of hadrons, nuclei and stars.

Despite the achievement of historical results, serious doubts have emerged in regard to the $exact$ character of quantum mechanics for $interior$ problems, such as [15,16]:

1) Quantum mechanics has permitted the achievement of a numerically $exact$ representation from first principles of all experimental data of exterior dynamical problems, By contrast, when passing to interior problems, quantum mechanics has only permitted an $approximate$ representation of experimental data, an occurrence that, per se, is a direct indication of the merely approximate character of quantum mechanics for interior conditions. For instance, quantum mechanics has provided an exact representation of the structure of the hydrogen atoms, while it misses 2% of the binding energy of the hydrogen molecule from unadulterated quantum principles [32]. In nuclear physics, quantum mechanics misses an

exact representation of the simplest possible nucleus, the deutneriom, since there are insufficiencies in the representation of its spin, magnetic moment, stability and other features, with dramatic insufficiencies for heavy nuclei such as the zirconium [16].

2) The No Reduction Theorems establish that the nonlinear, nonlocal and nonpotential character of our macroscopic systems originate at the ultimate level of elementary particles, thus requiring a covering of quantum mechanics. As an example, the approximate character of quantum mechanics for the hydrogen molecule originates from the conditions of deep mutual penetration of the wavepackets of the two valence electrons by characterizing interactions dramatically beyond a possible representation by quantum mechanics. Similar occurrences hold for other interior problems.

3) The representation of interior conditions via quantum mechanics is generally done with the use of completely arbitrary parameters or functions of unknown physical origin that are fitted from the data, and quantum mechanics is then claimed as being exactly valid. As an example, an exact representation of the binding energy of the hydrogen molecule is achieved via the so-called "screened Coulomb potentials," that is, the multiplication of the Coulomb potential by an arbitrary function such as $V(r) = f(r)e^2/r$, and then the fitting of the arbitrary function $(f(r)$ from the experimental data. However, it is known that "screened Coulomb potentials" do not admit quantized levels and, therefore, the very name "quantum chemistry" becomes questionable [32]. In particle physics, the use of $ad\ hoc$ parameters and functions for interior conditions has reached at time paradoxical characters. For instance, the experimental data of the two-points function of the Bose-Einstein correlation are fitted via the use of $four$ arbitrary parameters (called the "chaoticity parameters") and then the claim that relativistic quantum mechanics is exact. However, the quantum axioms for the expectation value of a two-dimensional Hermitean operator may admit, under debatable assumptions, a maximum of two arbitrary parameters, the use of four parameters being excluded by the very axioms of quantum mechanics [16a]. A deeper inspection has shown that the missing two parameters must originate from $off\text{-}diagonal\ elements$ in the vacuum expectation values thus casting shadow on the

consistent representation of observables.

In view of the above and numerous other insufficiencies [16a,24], a vast effort has been conducted by numerous scientists over decades for the construction of a nonlinear, nonlocal and nonpotential covering of classical and quantum mechanics known under the name of *hadronic mechanics* with the following main results [15,16,32,33]:

A) The construction of the so-called *iso-, geno-, and hyper mathematics* for the representation of variationally nonselfadjoint interior systems of *matter* that are single-valued reversible, single valued irreversible, and multi-valued irreversible, respectively, and their isoduals for *antimatter,* these new mathematics being characterized by different generalized units as outlined in Section 3;

B) The construction of corresponding new classical mechanics, known as *iso-, geno- and hyper-Lagrangian or Hamiltonian mechanics for matter,* and their isoduals for *antimatter,* achieving the representation of interior dynamical systems via an action principle, as outlined in paper II; and

C) The *isotopic, genotopic and hyperstructural branches of hadronic mechanics* for the operator representation of the above identified interior systems of *matter,* and their isoduals for *antimatter,* possessing progressively increasing complexity and methodological needs, as also outlined in paper II.

The above formulations have indeed allowed exact representations of interior problems from unadulterated first axioms, such as an exact representation of the binding energy and other features of the hydrogen molecule from first principles without arbitrary functions [32], an exact representation of the experimental data of the Bose-Einstein correlation from first principles without arbitrary parameters, and other interior problems in classical physics, particle physics, nuclear physics, supercondiuctivity, chemistry, astrophysics and cosmology (see Vol. [16d] and Chapter 5 of Ref. [24] for a review).

We are now equipped to formulate the following:

ASSUMPTION 2.3: Quantum mechanics is assumed as being exactly valid everywhere in the exterior of the scattering region, while the covering hadronic mechanics is assumed as being exactly valid in the interior region.

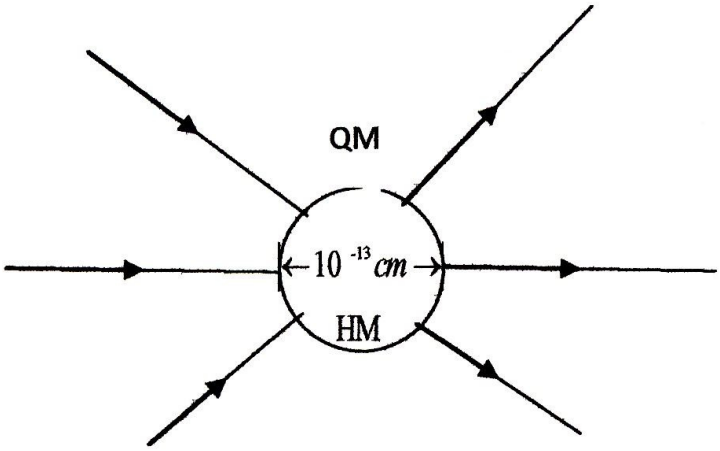

Figure 2: *A schematic view of the main assumptions of these papers, the validity of conventional quantum mechanics everywhere in exterior conditions, and the validity of the covering hadronic mechanics for interior conditions.*

The smooth transition from the interior (hadronic mechanics) to the exterior (quantum mechanics) is simply achieved via realizations of the generalzied unit of the type

$$Lim_{r>1fm}\hat{I}(t,t,r,p,\psi,...) = \hbar. \tag{2.1}$$

As we shall see in paper II, the above condition is quite naturally verified by all meaningful realizations of the generalized unit.

In view of the general inequivalence of $\hat{I}(t,r,p,\psi,...)$ and I, the evident lack of general commutativity of $\hat{I}(t,r,p,\psi,...)$ and $H(r,p)$, and other aspects, the isoscattering theory requires a reinspection of the data elaboration of experimental data achieved with the conventional scattering theory to ascertain whether said data elaborations persist under nonlinear, nonlocal and nonpotential internal effects, or the final numerical values themselves need a revision.

2.5. Restrictions for Irreversibility and Antimatter. Recall that the formalism of the covering scattering theory includes that of quantum

mechanics, plus three covering formalisms of hadronic mechanics with progressively increasing complexity, and all their isoduals for antimatter. To avoid the initiation of the study with excessive complexities, in these three papers we shall restrict our formulations to *isomathematics and isomechanics,* resulting in the suggested name of *isoscattering theory,* where the reader should keep in mind that the prefix "iso" indicates the preservation of the axioms of the conventional theory, and merely the use of *broader realizations.*

This restriction implies that, by conception and construction, *the isoscattering theory does not generally represent irreversible processes,* except under certain conditions, as we shall see, such as isounits that are Hermitean but time noninvariant

$$\hat{I}(t, r, p, \psi, ...) \neq \hat{I}(-t, r, p, \psi, ...). \tag{2.2}$$

In other words, we shall essentially study scattering processes in the way they are treated by quantum mechanics, without a quantitative representation of their irreversibility, and shall address the latter issue in a subsequent paper based on *Lie-admissible genomathematics and genomechanics* [25]. In any case, the construction of the Lie-isotopic isoscattering theory is a recommendable pre-requisite for the much broader *Lie-admissible irreversible genoscattering theory.*

Additionally, *the isoscattering theory of these first papers does not include antiparticles* also to avoid excessive complexities at start up. This additional restriction is due to recent advances in antimatter that have achieved full scientific democracy between matter and antimatter at all levels of study, from Newtonian mechanics to second quantization, thus ending the scientific imbalance of the 20th century of treating antimatter at the sole quantum or quantum field theoretical levels [33].

These advances have been stimulated by E. C. G. Stueckelberg conception of *antimatter with a negative time,* but the achievement of consistency required the use of a conjugation of all physical and mathematical quantities, thus leading to *negative time, energy, and other physical quantities referred to corresponding negative units,* that are as causal as conventional positive time, energy and other physical quantities referred to corresponding positive units.

The treatment of this new setting required the construction of the new *isodual mathematics* that is anti-isomorphic to conventional mathematics in all its parts and operations. In turn, these advances have identified a new symmetry, called *isoselfduality,* essentially given by invariance under anti-Hermiticity trivially verified by the imaginary unit $i = -i^\dagger$, but less trivially verified by the Dirac equation and related gamma matrices (see Ref. [33] for details)

$$\gamma_\mu \equiv -\gamma_\mu^\dagger, \tag{2.3}$$

and other cases.

Physically, isoselfduality has emerged as representing systems of particles and their anti[particles, thus permitting a new interpretation of the Dirac equation as providing a direct *quantum* representation of an electron and its antiparticle (the positron) without any need for the "hole theory," since the the isodual theory applies at the *classical,* let alone purely quantum level, where it reaches equivalence with the conventional charge conjugation [33].

Therefore, the inclusion of antiparticles in our study of scattering processes requires a reinspection of the very structure of the *conventional* Feynman's diagrams so as to achieve a full democracy of treatment between particles and antiparticles, thus suggesting a separate treatment to avoid excessive complexities at start up.

It should be noted, as we shall see in paper II., that *the invariance under isoselfduality is generally violated by quantum scattering treatments inclusive of particles and antiparticles.* This occurrence alone mandates a reinspection *ab initio* of scattering theories in general, let alone when including particles and antiparticles.

In these papers, we shall use the terms "quantum mathematics," "quantum scattering theory," etc. to denote aspects pertaining to quantum mechanics and use the terms "hadronic mathematics," "hadronic scattering theory," etc. to denote their corresponding coverings as characterized by hadronic mechanics.

A number of divergent terminologies exist in the literature of this paper as compared to that of the quantum scattering theory. For instance, the term "potential" is used in the literature of hadronic mechanics as a synonym of "Hamiltonian" or, more technically, referring to the verifica-

tion of all integrability conditions for the existence of a Hamiltonian [30], while systems of that class are not necessarily called "potential" in the quantum literature.

This is the case for the interaction term $H_1 = J * A$ that is generally considered as being of nonpotential character in the quantum literature, while it verifies the conditions of variational self-adjointness (see monographs [30]), thus being of a true potential for the hadronic literature, as confirmed in any case by the fact that said interaction term is fully "Hamiltonian" and *additive* to the kinetic term and other potentials, e. g., $H = H_0 + H_1$.

By comparison, the terms "nonpotential" is used in the hadronic literature to stress the impossibility of representing the novel "nonpotential" interactions with a Hamiltonian, technically referring to the *violation of the conditions of variational self-adjointness in the frame of the experimenter,* thus requiring new vistas.

3. Elements of Santilli Isomathematics.

3.1. Introduction. As indicated in Sections 1 and 2, numerous aspects warrant the broadening of the scattering theory to incorporate non-Hamiltonian effects, that is, effects that cannot be represented via the conventional Hamiltonian. Any meaningful broadening of the conventional scattering theory requires the existing from the class of unitary equivalence of quantum mechanics. However, the ensuring nonunitary theories are afflicted by a litany of problems known under the name of *Theorems of Catastrophic Inconsistencies of Nonunitary Theories* [6-12]. Consequently, the central objective of this section is to *identify an equivalent formulation of nonunitary theories resolving the inconsistency problems.*

Following decades of research, the solution of the above problem required the construction by various authors of a *new mathematics,* known as *isomathgematics,* originally proposed by Santilli [4] in 1978, subsequently studied by the same author in disparate works, as well as by numerous pure and applied mathematicians, including (in chronological order of contributions) R. M. Santilli, S. Okubo, H. C. Myung, M. L. Tomber, Gr. T. Tsagas, D. S. Sourlas, J. V. Kadeisvili, A. K. Aringazin,

A. Kirhukin, R. H. Ohemke, G. F. Wene, G. M. Benkart, J. M. Osborn, D. J. Britten, J. Lohmus, E. Paal, L. Sorgsepp, D. B. Lin, J. V. Voujouklis, P. Broadbridge, P. R. Chernooff, J. Sniatycku, S. Guiasu, E. Prugovecki, A. A. Sagle, C.-X. Jiang, R. M. Falcon Ganfornina, J. Nunez Valdes, A. Davvaz, and others (see the comprehesnive bibliography at the end of Ref. [16a]). To illustrate the complexity of the problems to be addressed, following the original proposal of 1978, initial mathematical maturity was solely achieved in memoir [13] of 1996, thus warranting this review and specialization to the scattering region so as to avoid possible insidious misinterpretations.

For the benefit of experimentalists we indicated that, as a result of the above efforts, the new mathematics can be constructed via the systematic application of $axiom\text{-}preserving\ liftings,$ called $isotopies,$ of the $totality$ of the mathematics of quantum mechanics, including all its operators and all its operations, thus including the isotopic lifting of numbers, functional analysis, differential calculus, geometries, topologies, Lie theory, symmetries, etc. [13,15,16]. As we shall see in paper II, said isotopies can be very easily constructed via the application of nonunitary transforms to the totality of the formalism of the conventiona;l scattering theory, thus being indeed accessible to experimentalists.

The physical needs for isomathematics have been indicated in Sections 1 and 2, and consists in the necessity for a representation of non-Hamiltonian scattering effects in a form that is $invariant\ over\ time$ so as to admit the sam,e numerical predictions under the same conditions at different times. Following the study of all possible alternatives, the latter condition required the representation of non-Hamiltonian scattering effects with an axiom-preserving generalization of the trivial (positive-definite) unit of quantum mechanics $\hbar = 1$ into the most general possible (positive-definite as a condition to characterize an isotopy), integro-differential operator $\hat{I}$. Since the unit is the fundamental (left and right) invariant of any theory, whether conventional or generalized, the representation of non-Hamiltonian effects via the isounit has indeed achieved the desired time invariant representation.

However, the assumption of a generalized unit has requested the compatible reconstruction of the entire mathematics used in quantum me-

chanics with no exception known to the authors. In fact, the sole elaboration of the isoscattering theory, e.g., with conventional trigonometric functions, activates the Theorems of Catastrophic Inconsistencies because it would be the same as elaborating the conventional scattering theory, e.g., with isotrigonometric functions.

Since no formulation of isomathematics specialized intended for scattering problems has been presented to date, it is important to outline it in this first paper for minimal self-sufficiency of the presentation, as well as to minimize possible insidious misinterpretations that may be caused by insufficient technical knowledge of the field. In this section we shall outline the rudiments of isomathematics for a positive-definite but otherwise arbitrary isounit $\hat{I}$ and show the resolution of the inconsistency problem under isotopies.

We should also indicate the distinction between *deformations* and *isotopies.* The former are alterations of conventional quantum formulations defined over conventional fields, thus being catastrophically inconsistent on mathematical and physical grounds (see Refs. [6-11] for brevity), while the latter can be characterized as deformations defined over isofields, thus avoiding the inconsistency theorems.

Note that isofields were introduced in 1993 [12]. Consequently, the contemporary formulation of deformations coincide with previously proposed isotopies, as it is the case for the *isotopies of the Lorentz symmetry* first proposed by Santilli in 1983 [34], at that time, over conventional fields, and subsequently reintroduced identically, even in the symbols and terms, as deformations, unfortunately, without the quotation of the original derivation [34]. Similar occurrence hold for other deformations 9see Ref. [15a] for brevity).

In these papers, conventional terms, such as numbers, spaces, etc. are referred to conventional notions of quantum mathematics. The corresponding notions of hadronic mathematics are indicated isonumbers, isospaces, etc. We regret a perhaps excessive use of the prefix "iso," but it appears recommendable in a first presentation of *applied mathematics* to prevent insidious inconsistencies.

Within the context of *pure mathematics,* we shall show that *both the conventional and the isotopic mathematics can be presented with the same*

symbols and operations, since they coincide at the abstract level by conception and construction. However, the latter formulation requires, in any case, an in depth knowledge of the isotopic realization of conventional abstract axioms, thus warranting again the use of the prefix "iso" in this first presentation, with the understanding that pure mathematicians may subsequently achieve the necessary mathematical rigor.

It is at times indicated that, due to the above abstract identity, isomathematics is trivial, a view perhaps correct. but only following its discovery. However, the implications solely permitted by isomathematics. such as the extension of Lie's theory, the Lorentz-Poincaré symmetry and Einstein's axioms for the treatment of nonlinear, nonlocal and non-Hamiltonian systems, are far from being trivial.

3.2. Isounits, Isoproducts and Isofields. As indicated earlier, isomathematics is based on the following isotopic, thus axiom-preserving lifting of the trivial unit into the most general possible positive-definite integro-differential operator

$$\hbar = 1 > 0 \to \hat{I}(t, r, p, E, \xi, \omega, \psi, \partial\psi, ...) = 1/\hat{T}(t, r, p, E, \xi, \omega, \psi, \partial\psi, ...) > 0. \tag{3.1}$$

first introduced in 1978 [4,5] and known as *Santilli isounit,* while $\hat{T}$ is known as the *isotopic element.* We shall use the notation T when the isotopic element is projected on quantum spaces, but keep the notation $\hat{I}$ to avoid confusion with I.

The isotopic lifting of the (multiplicative) unit evidently requires a corresponding compatible lifting of *all* multiplications between arbitrary quantities A, B, from the simple associative form used in quantum mechanics, herein denoted $AB = A \times B$, to the new form first introduced by Santilli in Ref. [4] of 1978

$$AB = A \times B \to A\hat{\times}B = A \times T \times B, \tag{3.2}$$

which is also isotopic, because it verifies the associativity law of the original product. It is easy to see that, under lifting (3.2), $\hat{I}$ is indeed the correct left and right unit of the theory, $\hat{I}\hat{\times}\hat{A} = A\hat{\times}\hat{I} = A$ for all elements A of the set considered.

Fundamental assumptions (3.1) and (3.2) have permitted the isotopic lifting of numerical fields $F(a, \times, I)$, such as the field of real numbers $R(n, \times, I)$, complex numbers $C(c, \times, I)$ and quaternions $Q(q, \times, I)$ into the *Santilli isofields* $\hat{F}(\hat{a}, \hat{\times}, \hat{I})$ [12], consisting of the original numbers $a = n, c, q$ lifted into the form of *Santilli isonumbers* $\hat{n} = n \times \hat{I}$ equipped with isounit (3.1) and isoproduct (3.2), $\hat{n}_1 \hat{\times} \hat{n}_2 = (n_1 \times n_2) \times \hat{I}$, as well as with the *conventional* sum $\hat{n}_1 \hat{+} \hat{n}_2 = \hat{n}_1 + \hat{n}_2$ and related conventional additive unit 0, $\hat{n} + \hat{0} = \hat{0} + \hat{n} = \hat{n}$, i.e., $\hat{0} = 0 \times \hat{I} \equiv 0$.

To avoid inconsistencies, it should be stressed that *all* operations with numbers have to be lifted in an isotopic form we cannot possibly review here (see [15]). We merely mention for use in the isoscattering theory the *isodivision* given by $\hat{/} = / \times \hat{I}$ so that we have simplifications in isomultiplications of the type $(a \hat{/} b) \hat{\times} (c \hat{/} d) = [(a/b) \times (c/d) \times \hat{I}$.

Also, and very importantly, conventional numbers expressing numerical values of physical quantities such as coordinates $r.$, momenta p, energy E, etc. have no meaning for isomathematics and must be lifted into the isotopic form $\hat{r} = r \times \hat{I}, \hat{p} = p \times \hat{I}, \hat{E} = E \times \hat{I}$, etc. as a necessary condition to be elements of a Santilli isofield, that is, to be *isoscalars.*

Readers should, however, be reassured that conventional numbers, as needed for experiments, are indeed recovered by the isoscattering theories. As an example, the (right, modular, associative) eigenvalue expression $E \times |\psi >$ becomes for isomathematics $\hat{E} \hat{\times} |\hat{\psi} >$ that can be simplified in the form $E \times \hat{I} \times \hat{T} \times |\hat{\psi} >= E \times |\hat{\psi} >$, thus recovering the conventional real number E needed for measurements.

It should be indicated that *isofields are isomorphic to ordinary fields, by conception and construction,* a property necessary for the consistent application of the isoscattering theory to experimental measurements. In fact, Santilli merely provided in Ref. [12] a *broader realization* of the conventional field axioms. The nontriviality of the realization is indicated by the fact that *the isounit of a Santilli isofield* $\hat{F}(\hat{a}, \hat{\times}, \hat{I})$ *is generally outside the original field* $F(a, \times, I)$. In this case, $\hat{F}(\hat{a}, \hat{\times}, \hat{I})$ are called *isofields of the first type.* When $\hat{I} \in F$, we have *isofields of the second type.*

Despite the simplicity of the isonumber theory, readers should be warned against predictable perceptions of triviality because, for instance,

under the assumption of the isounit $\hat{I} == 3$, thus dealing with isofields of the second type, we have "2×3"$= 18$ and the *number* 4 *becomes a prime number.*

For in depth knowledge of Santilli isofield theory and its intriguing implications, interested readers are suggested to study the original paper [12], Ref. [15a] and Jiang's monograph [22].

3.3. Isofunctional Analysis. Any elaboration of the isoscattering theory with conventional functions, such as sine, cosine, exponential, etc. leads to inconsistencies [6-11,15]. Even though not clearly indicated in the mathematical literature, all functions crucially depend on the assumed basic unit and multiplication. Therefore, liftings (3.1) and (3.2) have required the laborious reconstruction of functional analysis into a form compatible with the basic axioms of isomathematics.

Studies on the *isofunctional analysis* were initiated by Santilli [4] and continued by Myung and Santilli [26], Kadeisvili [21], Nishioka [27] Aringazin [29] and others (see the general bibliography of Ref. [16a] for a comprehensive listing). A presentation of isofunctional analysis sufficient for the isoscattering theory is available in monograph [15a]. For completeness we recall the following notions:

3.3.1) *Isopowers,*

$$\hat{a}^{\hat{n}} = \hat{a}\hat{\times}\hat{a}\hat{\times}...\hat{a} = (a^n) \times \hat{I}, \tag{3.3}$$

for which $\hat{I}^{\hat{n}} = \hat{I}$;

3.3.2) *Isoexponentiation,*

$$\hat{e}^a = \hat{I} + a\hat{/}\hat{1}! + a\hat{\times}a\hat{/}\hat{2}! + ... = (e^{a\times T}) \times \hat{I} = \hat{I} \times (e^{T\times a}), \tag{3.4}$$

where one should note the emergence of the integro-differential quantity T in the *exponent;*

3.3.3) *Isologarithm,*

$$\hat{log}_{\hat{e}}\hat{a} = \hat{I} \times log_{\hat{e}}a, \tag{3.5}$$

which expression is indeed the inverse of the isoexponentiation, as one can verify, as well as yields a correct isonumber for result;

3.3.4) *Isotrigonometric functions* (for isospherical coordinates see later on Section 3.8),

$$\hat{sin}\hat{\theta} = T_\theta \times sin(\theta \times \hat{I}_\theta), \tag{3.6a}$$

$$\hat{cos}\hat{\phi} = T_\phi \times cos(\phi \times \hat{I}_\phi, \tag{3.6b}$$

where evidently the isounits for angles are generally different than those for space.

Note that the use of conventional angles would have no sense for the isoscattering theory because all numbers must be isonumbers for consistency. We shall identify later on specific realizations of the various isounits.

A rather intriguing and unexpected feature of isotopies is that of preserving on isospaces over isofields the *numerical values* of the quantities prior to lifting. This feature has been crucial for the reconstruction of the exact light cone and special relativity on isospace over isofield when light becomes a local variables, thus requiring in conventional spaces deformed light cones.

According to this feature, the isoscattering theory is expected to preserve the numerical value of the angles θ and ϕ as measured in experiments. However, the preservation is for the new isoangles $\hat{\theta}$ and $\hat{\phi}$. Consequently, the correct identification is

$$\theta = \hat{\theta} = \theta' \times \hat{I}_\theta, \;\; \phi = \hat{\phi} = \phi' \times \hat{I}_\phi. \tag{3.7}$$

The above rules indicate the expected differences in the elaboration of experiments via the scattering and isoscattering theories.

3.3.5) *Isomatrices*, given by conventional matrices whose elements are isoscalars, such as for the diagonal case

$$\hat{M} = Diag.(\hat{a}_1, \hat{a}_2, ..., \hat{a}_n), \tag{3.8}$$

where $\hat{a}_k = a_k \times \hat{I}$;

3.3.6) *Isodeterminant*,

$$\hat{Det}\hat{M} = [Det(\hat{M} \times \hat{T}) \times \hat{I}, \tag{3.9}$$

where one should note that $\hat{M} \times \hat{T}$ is an ordinary matrix. Hence, the value of the isodeterminant is indeed an isonumber.

3.3.7) *Isotrace,*

$$\hat{Tr}\hat{M} = Tr(\hat{M} \times \hat{T}) \times \hat{I}, \tag{3.10}$$

etc. It should be stressed that the above elements of isofunctional analysis are merely introductory and a study of at least Chapter 6 of monograph [15a] is necessary for a serious knowledge of the isoscattering theory.

3.4. Isodifferential Calculus. It was believed for centuries that the differential calculus is independent of the assumed basic unit, since the latter was traditionally given by the trivial number 1.

Santilli [13] has disproved this belief by showing that the differential calculus can be dependent on the assumed unit, by formulating the *isodifferential calculus* with basic *isodifferential,* for instance, of an isocoordinate $\hat{r}$

$$\hat{d}\hat{r} = \hat{d}[r \times \hat{I}(r, ...)] = \hat{T} \times d[r \times \hat{I}(r, ...)], \tag{3.11}$$

that does indeed coincide with the conventional differential for all isounits independent from r, $\hat{d}\hat{r} \equiv dr$, while yielding structural differences for all cases relevant for the isoscattering theory, namely, when the isounit depends on the local coordinates. In the latter case we have

$$\hat{d}\hat{r} = T \times d[r \times \hat{I}(r, ...)] = dr + r \times T \times d\hat{I}(r, ...). \tag{3.12}$$

The compatible formulation of the *isoderivative* is then given by

$$\frac{\hat{\partial}}{\hat{\partial}\hat{r}} = \hat{I} \times \frac{\partial}{\partial \hat{r}}. \tag{3.13}$$

The *isointegral* is defined as the inverse of the isodifferential and can be written for simplicity

$$\hat{\int} \hat{d}\hat{r} = \int d\hat{r}, \quad \hat{\int} \hat{d}\hat{r} \hat{\times} \hat{f}(\hat{r}) = \int d\hat{r} \times f(\hat{r}), \tag{3.14}$$

where we have used the isofunction $\hat{f}(\hat{r}() = \hat{I} \times f(\hat{r})$.

Note that, as formulated above for simplicity, isodifferentiation and isointegration yield ordinary scalars and *not* isoscalars, a feature assumed

later on in Section 2.3 of paper II to reach a formulation accessible to experimentalists.

It should be indicated that the use of the conventional differential calculus leads to catastrophic mathematical and physical inconsistencies particularly in the dynamical equations [6-11], thus mandating the use of the covering isodifferential calculus. Consequently, the sole functional differences between the conventional and isodifferential calculus are sufficient to warrant a reinspection of the quantum scattering theory.

As an illustration, the realizations of the isounit of primary physical relevance are based on exponentials, e.g., $\hat{I} = \hat{M}\hat{\times}exp[f(r,...)]$, where $\hat{M}$ is a matrix or operator not dependent on r. In this case, the isounit and the isotopic element disappear from the projection of the isodifferential in our space. This results in significant deviations between conventional and isotopic differentials, e.g., $dr \neq \hat{dr} = dr \times (1 + r \times \partial f \partial r)$ thus providing additional expectations of possible numerical differences in the final elaboration of the same experiment with the conventional and the isotopic scattering theory.

3.5 Iso-Hilbert Spaces. The fundamental representation space of hadronic mechanics is a *new realization* of the abstract axioms of the conventional Hilbert space $\mathcal{H}$ over the field of complex numbers $\mathcal{C}$, first proposed by Santilli [5] in 1978, then studied by Myung and Santilli [26] and other authors (see the review in Ref. [15a] and quoted references), today known as *iso-Hilbert spacve* or *Hilbert-Myung-Santilli isospaces,* and denoted $\hat{\mathcal{H}}$ over the isofield $\hat{\mathcal{C}}$. The new space is characterized by *isostates* $|\hat{\psi}>$ with *isoinner product,* and related *isonormalization,*

$$< \hat{\psi}|\hat{\times}|\hat{\psi} > \times\hat{I} =< \hat{\psi}| \times \hat{T} \times |\hat{\psi} > \times\hat{I}\, \epsilon\, \hat{C}, \tag{3.15a}$$

$$< \hat{\psi}|\hat{\times}|\hat{\psi} > \times\hat{I} = \hat{I}, \tag{3.15b}$$

isoexpectation values of an iso-Hermitean operator $\hat{Q} = \hat{Q}^{\hat{\dagger}}$

$$< \hat{\psi}|\hat{\times}\hat{Q}\hat{\times}|\hat{\psi} > \times\hat{I} =< \hat{\psi}| \times T \times Q \times T \times |\hat{\psi} > \times\hat{I}, \tag{3.16}$$

isounit under isonormalization (3.16)

$$< \hat{\psi}|\hat{\times}\hat{I}\hat{\times}|\hat{\psi} > \times\hat{I} =< \hat{\psi}| \times \hat{T} \times \hat{T}^{-1} \times |\hat{\psi} > \times\hat{I} = \hat{I}, \tag{3.17a}$$

$$\hat{I}\hat{\times}|\hat{\psi}>\equiv|\hat{\psi}>; \tag{3.17b}$$

isoeigenvalue equation for iso-Hermitean operators

$$\hat{H}\hat{\times}|\hat{\psi}>=H\times T\times|\hat{\psi}>=\hat{E}\hat{\times}|\hat{\psi}>=E\times|\hat{\psi}>,\ \hat{E}\in\hat{\mathcal{R}},\ E\in\mathcal{R}; \tag{3.18}$$

and additional properties we cannot possibly review here. We limit ourselves to quote the following main properties (see Ref. [15a] for details):

3.5.1) Hilbert-Santilli isospaces are isomorphic to conventional Hilbert spaces by conception and construction, as illustrated by the fact that the isoinner product (3.15) is still inner from the positive-definite character of the isounit. This property is crucial to ensure the covering character of hadronic over quantum mechanics, as well as the existence of a unique and unambiguous interconnecting maps indicated below.

3.5.2) Operators that are Hermitean on $\mathcal{H}$ over $\mathcal{C}$ are also iso-Hermitean, namely, they remain hermitean under lifting to the Hilbert-Santilli isospace over the isofield of isocomplex numbers, and we shall often write $\hat{Q}=\hat{Q}^{\hat{\dagger}}=\hat{Q}^{\dagger}$. Therefore, all quantities that are observable for quantum mechanics remain observable for hadronic mechanics, although the opposite is not generally true because of the existence of Hermitean operators representing irreversible process that are well defined for hadronic mechanics but cannot be even formulated for quantum mechanics due to its simpler structure.

3.5.3) The conventional Hilbert space admits a new symmetry discovered by Santilli [13,14] called *isoscalar symmetry,* given by a rescaling of the unit under which the conventional inner product is invariant,

$$<\psi|\times|\psi>\times I\equiv$$

$$\equiv\ <\psi|\times w^{-1}\times|\psi>\times(w\times I)=<\psi|\hat{\times}|\psi>\times\hat{I},\ \ w\,\epsilon\,\mathcal{C}. \tag{3.19}$$

Evidently, the property persists for the Hilbert-Santilli isospace and we have

$$<\hat{\psi}|\hat{\times}|\hat{\psi}>\times\hat{I}=<\hat{\psi}|\times T\times|\hat{\psi}>\times\hat{I}\equiv$$

$$\equiv<\hat{\psi}|\times(w^{-1}\times T)\times|\hat{\psi}>\times(w\times\hat{I})=<\hat{\psi}|\times T'\times|\hat{\psi}>\times\hat{I}'. \tag{3.20}$$

The lack of discovery of symmetry (3.19) for over one century should not be surprising, because the new symmetry required the prior discovery of new numbers, those with arbitrary units [12]. In fact, isosymmetry (3.19) requires the reformulation of numbers as isonumbers $\hat{n} = n \times 1$.

Despite its apparent triviality, the discovery of isosymmetry (3.19) has permitted the achievement of a new grand unification of gravitational and electroweak interactions essentially based on the embedding of gravitation where nobody looked for, in the *unit* of electroweak theories. The new grand unification includes the first known axiomatically correct inclusion of antimatter in grand unified theories also nobody cared for since gravitation on a Riemannian space cannot represent neutral antimatter. This suggests the use of the isodual theory of antimatter to achieve a grand unification with a degree of democracy between matter and antimatter (see papers [44-46] for original words and monograph [33] for comprehensive treatment).

3.6. Isolinearity, Isolocality and Isounitarity. We are now equipped to introduce the following important notions first introduced by Santilli as the foundation of the isotopies of Lie's Theory (see, e.g., Ref. [15]):

DEFINITION 3.6.1: ISOLINEARITY.
Operators that are nonlinear on $\mathcal{H}$ over $\mathcal{C}$ (that is, *nonlinear in the wavefunction*) can be *identically* rewritten in a form that is linear on $\hat{\mathcal{H}}$ over $\hat{\mathcal{C}}$, a property called *isolinearity.* The reformulation is simply done by embedding all nonlinear terms in the isounit, In fact. hadronic mechanics was proposed [5] to reformulate complex nonlinear models, e.g., $H(r,p,\psi) \times |\psi >= E \times |\psi >$, into an identical isolinear form $H_o(r,p) \times T(r,p,\psi) \times |\psi >= E \times |\psi >,\ H = H_o \times T$. Despite its simplicity, the reformulation is not trivial because the conventional nonlinear formulation generally violates the superposition principle, thus being generally inapplicable to composite systems, while the isotopic formulation verifies the superposition principle on isospace over isofield, thus allowing consistent studies of nonlinear composite systems.

DEFINITION 5.6.2: ISOLOCALITY.
Operators that are nonlocal on $\mathcal{H}$ over $\mathcal{C}$, e.g., of nonlocal-integral type,

can be *identically* reformulated in a form on $\hat{\mathcal{H}}$ over $\hat{\mathcal{C}}$ that is local-differential everywhere except at the isounit, a property known as *isolocality.* Again, the reformulation is done via the embedding of all nonlocal terms in the isounit. It should be noted that the technical understanding of isolocality requires a technical knowledge of the *isotopology* of hadronic mechanics initiated by the mathematicians Gr. Tsagas and D. S. Sourlas [34] (see also monograph [19]) and completed by the mathematicians M. Falcon Ganfornina and J. Nunez Valdes [35] (see also monograph [23]).

DEFINITION 5.6.3: ISOUNITARITY.
All operators U that are nonunitary on $\mathcal{H}$ over $\mathcal{C}$ can be *identically* reformulated in a form verifying unitarity on $\hat{\mathcal{H}}$ over $\hat{\mathcal{C}}$, a property called *isounitarity.* The reformulation is done via the simple identity

$$U \times U^\dagger \neq I, \;\; U = \hat{U} \times \hat{T}^{1/2}, \tag{3.21}$$

under which we have the *isounitarity law*

$$\hat{U} \hat{\times} \hat{U}^\dagger = \hat{U}^\dagger \hat{\times} \hat{U} = \hat{I}. \tag{3.22}$$

This is the property indicated in Section 1 that assures nonunitary S-matrices to preserve probabilities under the condition that the matrices are not treated via the mathematics of quantum mechanics.

3.7. Resolution of the Inconsistency Theorems. We are now sufficiently equipped to show the resolution of the Theorems of catastrophic Inconsistencies of Nonunitary Theories [6-11], first achieved by Santilli thanks to his isomathematics (see Ref. [15] for detailed treatment):

INVARIANCE OF THE BASIC UNIT.
The units of the conventional scattering theory characterize a geometrization of basic unfits of measurements. For instance, the unit of the three-dimensional Euclidean space is a geometrization of the units of length along each axis, e.g., $I = Diag(1cm, 1cm, 1cm)$. When expressed in dimensionless form, the unit acquires the familiar version $I = Diag.(1, 1, 1)$. All quantum units are invariant under unitary time evolution, $I \to U \times I \times U^\dagger \equiv I$, thus confirming the majestic axiomatic consistency of quantum mechanics.

However, these units are no longer invariant under nonunitary time evolutions $U \times U^\dagger \neq I$ because, in this case, we can have maps of the type $I \rightarrow U \times I \times U^\dagger = Diag.(231cm, 1.36cm, 0.3cm) \neq I$. This illustrates a first inconsistency of nonunitary scattering theories, the lack of preservation over time of the basic units of measurements, with consequential lack of consistent applicability of nonunitary theories to experiments.

A central features of the isoscattering theory is the invariance of the isounits $\hat{I}$ under the isounitary time evolution of the theory. In fact, under isounitarity law (22) we have, for instance, the invariance $\hat{I} = Diag.(231cm, 1.36cm, 0.3cm) \rightarrow \hat{U}\hat{\times}\hat{I}\hat{\times}\hat{U}^\dagger \equiv \hat{I}$, thus resolving the first inconsistency of nonunitary theories

INVARIANCE OF OBSERVABLES.
Another central property of quantum mechanics is that, when a quantity is observable at a given time, it remains observable at all subsequent times. This feature is verified by the preservation of Hermiticity under unitary time evolutions and provides another illustration of the majestic consistency of quantum mechanics.

When the time evolution is no longer unitary, Hermiticity is not necessarily preserved over time (this is the *Lopez lemma* [6] indicated in Section 1). In fact, the transformed eigenvalue equation for an operator H that is Hermitean at the initial time t_o under nonunitary transforms $U = U(t)$ is given by $H \times |\psi > \rightarrow (U \times H \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times |\psi > \times U^\dagger)$. Consequently, the initial Hermiticity of H is not necessarily preserved over time due to the lack of general commutativity of $U \times H \times U^\dagger$ and $(U \times U^\dagger)^{-1}$.

It is an instructive exercise for the reader interested in acquiring a knowledge of the isoscattering theory to prove that iso-Hermiticity is indeed preserved under isounitary transformations [6,12].

INVARIANCE OF NUMERICAL PREDICTIONS.
Yet another important feature of the axiomatic consistency of quantum mechanics is that, if a Hermitean operator H has the eigenvalue E (e.g., $E = 5MeV$) at the initial time, $H \times |\psi >= E \times |\psi >$, said eigenvalue is preserved at all times, as shown by the transformation $(U \times H \times U^\dagger) \times (U \times |\psi > \times U^\dagger) = H' \times |\psi' >= U \times (E \times |\psi > \times U^|dag = E \times |\psi' >$.

Under nonunitary time evolutions, the eigenvalue at the initial time of

a Hermitean operator is not necessarily preserved at subsequent times, as shown by the transformation $(U \times H \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times |\psi> \times U^\dagger) = H' \times T \times |\psi'> = U \times (E' \times |\psi> \times U^d ag) = E' \times |\psi'>, T = (U \times U^\dagger)^{-1}$, where the lack of preservation of the eigenvalue, $E' \neq E$, follows from the fact that $|\psi'>$ is now the eigenstate of the new operator $H' \times T$. It is an instructive exercise for interested readers to verify that isoeigenvalues are indeed preserved under isounitary time evolutions. The resolution of the remaining inconsistencies then follows [16a,16c].

The property important for the isoscattering theory is that *eigenvalues of Hermitean operators are numerically altered under nonunitary-isounitary lifting.* This occurrence suggests, alone, a reinspection of the conventional scattering theory because the possible presence of nonunitary effects in deep inelastic scattering could imply numerical results different than those currently assumed.

3.8. Delta Isofunction. As well known, Dirac's delta function, here expressed for the case of a one-dimensional coordinate r,

$$\delta(r - r_0) = \frac{1}{2\pi} \times \int_{-\infty}^{+\infty} e^{i \times k \times (r - r_o)} \times dk, \tag{3.23}$$

is divergent at $r = r_0$, by therefore constituting the origin of divergences in quantum scattering theories [1].

In view of the above, Myung and Santilli [26] introduced in 1982 the isotopic lifting of Dirac's delta function, today known as the *Dirac-Myung-Santilli delta isofunction,* or *DMS isodelta* for brevity (see, e.g., Nishioka [27]) that, by using the notions of isointegral (3.14), and isoexponentiation (3.4), can be written

$$\hat{\delta}(r - r_0) = \frac{\hat{I}}{2\pi} \hat{\times} \hat{\int}_{-\infty}^{+\infty} \hat{e}^{i \times k \times (r - r_0)} \hat{\times} \hat{dk}, = \frac{1}{2\pi} \times \int_{-\infty}^{+\infty} e^{i \times k \times T \times (r - r_o)} \times dk, \tag{3.24a}$$

$$T = \Sigma_{k=1}^{n} c_k \times (r - r_0)^k, \;\; c_k \in \mathcal{C}. \tag{3.24b}$$

where we write the isotopic element T without a "hat" to denote its formulation on conventional spaces, and example (3.24b) an illustration of

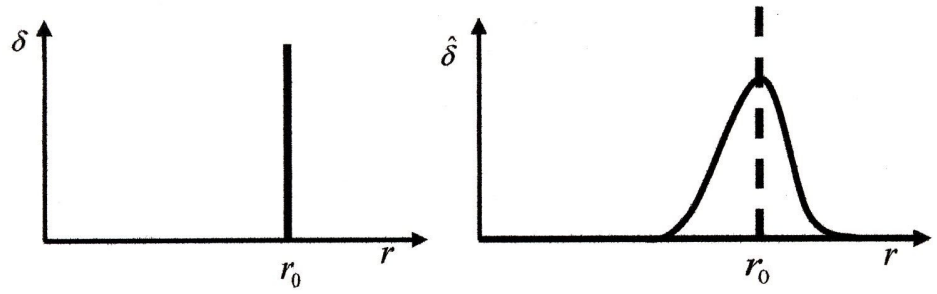

Figure 3: *A schematic view in the left of the conventional delta function* $\delta(r-r_0)$ *illustrating its divergent character at* r_0*, and a schematic view in the right of the Dirac-Myung-Santilli isodelta function of hadronic mechanics* $\hat{\delta}(r-r_0) = \delta[T(r-r_0)]$*, illustrating the absence of the above divergency, a feature allowing the removal of divergencies in the isoscattering theory from primitive axioms.*

the possible removal of the singularity at r_0. We then have the evident property

$$\hat{\delta}(r-r_0) = \delta[T \times (r-r_0)]. \tag{3.25}$$

As illustrated in Figure 2, under the appropriate realization of the isotopic element T, the DMS isodelta eliminates the divergent character of the delta function, thus setting up the foundations for a new scattering theory without divergencies *ab initio,* which is a main objective of this paper.

Note that for (3+1)-dimensional spaces each coordinate is multiplied by its isotopic element (see next section). For numerous additional properties, e.g., the derivation of the isodelta via isotransforms, the reader is encouraged to study monograph [xx]. Section 6.4.

3.9. Isospherical Isocoordinates. An additional mathematical notion needed for the elaboration of the isoscattering theory is given by the *isospherical coordinates* [15] here considered for in the Euclid-Santilli

isospace with isounit

$$\hat{I} = Diag.(/b_1^2, 1/b_2^2, 1/b_3^2) = 1/T > 0, \tag{3.26}$$

isometric

$$\hat{\delta} = T \times \delta = Diag.(b_1^2, b_2^2, b_3^2), \tag{3.27}$$

and isoinvariant

$$\hat{r^2} = x^2 \times b_1^2 + y^2 \times b_2^2 + z^2 \times b_3^2. \tag{3.28}$$

Under the assumption of the conventional orientation of the angles θ, ϕ with respect to the z-axis, we have the isounits

$$\hat{I}_\theta = b_3, \;\; \hat{I}_\phi = b_1 \times b_2, \tag{3.29}$$

and the projection of the isocoordinates on the conventional Euclidean space

$$x = r \times b_1^{-1} \times sin(\theta \times b_3) \times cos(\phi \times b_1 \times b_2), \tag{3.30}$$

$$y = r \times b_2^{-1} \times sin(\theta \times b_3) \times sin(\psi \times b_1 \times b_2), \tag{3.31}$$

$$z = r \times b_3^{-1} \times cos(\theta \times b_3). \tag{3.32}$$

Understanding of the isoscattering theory requires the knowledge that Eq. (3.28) represents an ellipsoid only when considered on the Euclidean space with respect to the trivial unit 1, because the same invariant represents the perfect sphere in Euclid-Santilli isospace over isofield called $isosphere.$ This is due to the fact that the k-axis is mutated by the quantity $1 \to b_k^2$, but the corresponding unit is mutated by the $inverse$ amount $1 \to b_k^{-2}$, thus preserving the perfect sphericity.

Similarly, the rotational symmetry has been popularly believed in the 20th century as being broken for ellipsoid (3.41), while in reality such a breaking is due to insufficient treatment since the rotational symmetry is reconstructed as exact on Euclid-Santilli isospaces, as shown by the perfect sphericity of the isosphere.

4. Concluding Remarks

In this paper, we have suggested the re-inspection of the conventional, potential, unitary scattering theory of relativistic quantum mechanics on grounds of the following aspects:

1) The apparent inapplicability (rather than violation) of the Lorentz-Poincaré symmetry and special relativity within physical media at large, and within the scattering region in particular, due to: difficulties for a consistent formulation of their axioms (impossibility of introducing inertial systems within a medium, the sole existence of the privileged reference frame at rest with the medium, difficulties in the verification of all axioms within a transparent medium, and others); deviations predicted in the repetition within physical media of the historical experimental verifications of special relativity in vacuum (repetition of Fizeau experiment entirely within water, and others); difficulties in reaching a $numerical$ (rather than solely conceptual) representation of all data for all frequencies in the $entire$ reduction to photons of electromagnetic waves propagating within physical media (inability to reach a numerical representation of the angle of refraction and the index of refraction; impossibility for a large number of photons to pass through a large number of nuclei as needed to maintain the main nonscattered part of e light beam along a straight line; difficulties in reducing to photons electromagnetic waves with one meter wavelength propagating within physical media; impossibility of representing with photons traveling in vacuum seemingly unavoidable superluminal causal speeds within physical media; etc.); and other insufficiencies;

2) Impossibility of reducing to photons traveling in vacuum the electromagnetic phenomena within the scattering region due to its hyperdense character, thus implying the locally varying speed $C = c/n$, suggesting a return to the Maxwellian interpretation of light and photon wavepackets as transversal electromagnetic waves propagating in the ether as a universal substratum without conflict with special relativity in vacuum (due to our impossibility of identifying a privileged system at rest with the ether), and consequential relevance of the $Lorentz\ problem,$ namely. the achievement of the universal symmetry for all locally varying speeds of light $C = c/n$;

3) The strict $reversibility$ over time of the Lorentz-Poincaré symme-

try and special relativity compared to the strict *irreversibility* over time of high energy inelastic scattering processes, with ensuing difficulties for rigorous verifications of causality and other laws, and the need for covering theories as irreversible as the scattering process being represented;

4) The need advocated by Heisenberg for a covering of quantum mechanics which is nonlinear in the wavefunction and other quantities due to the expected nonlinearity of high energy scattering processes, compared to the linear character of quantum mechanics, the breaking of the superposition principle for Hamiltonians dependent on wavefunctions and consequential inapplicability of nonlinear quantum models to composite scattering processes;

5) Einstein-Podolsky-Rosen historical doubts on the final character of quantum mechanics; Dirac's call for a reformulation of the scattering theory that is convergent *ab initio* so as to avoid the achievement of numerical results in high energy scattering experiments via *ad hoc* procedures to achieve mathematical convergence of unknown physical origin or content; and other authoritative doubts;

6) The *No Reduction Theorems* preventing a consistent reduction of macroscopic irreversible systems to a finite set of particles all in nice conservative conditions, with consequential impossibility of reducing highly irreversible scattering processes to point-like quantum particles verifying the rotational and Lorentz symmetries, thus identifying the origin of irreversibility in the total mutual penetrations of the wavepackets and/or charge distributions of particles in the scattering region, essentially as occurring for macroscopic irreversible systems (such as a spaceship during reentry in atmosphere);

7) The unavoidable non-Hamiltonian and, therefore, nonunitary character of the contact effects due to total mutual penetration of extended wavepackets and/or charge distributions of particles in the scattering region, with consequential exiting from the class of unitary equivalence of quantum mechanics;

8) The numerical alteration of the eigenvalues of scattering operators under non-Hamiltonian, thus nonunitary internal effects, with consequential possible lack of final characer of the data elaboration of measured quantities (cross sections, scattering angles, etc.) via unitary scat-

tering theory;

9) The recent discovery of the invariance of particle-antiparticle systems under the new symmetry called *isoselfduality* (invariance under anti-Hermiticity) that is verified by the Dirac equation, resulting in its direct representation of an electron and a positron without need for the "hole theory," said new invariance not being generally verified by the scattering amplitude for particle-antiparticle processes;

and other aspects all concurring in a return to the old need for a nonunitary covering of the conventional unitary scattering theory.

In this paper, we have then recalled the *Theorems of Catastrophic Mathematical and Physical Inconsistencies of Noncanonical and Nonunitary Theories,* implying the lack of invariance over time of the units of measurements, the lack of conservation over time of observable, the general inability to predict the same numerical results under the same conditions at different times, and others serious insufficiencies.

In order to avoid excessive complexities at start up, in this and the following papers we have restricted our analysis to *reversible* scattering processes *without antiparticles.* We have then, apparently for the first time, specialized to the scattering region the new mathematics known as *isomathematics,* that has been specifically built over decades of efforts by various authors to bypass said inconsistency theorems; we have outlined their resolution; and restricted the study to a time reversal invariant formulation of the nonunitary scattering theory without antiparticles under the name of *isoscattering theory.*

In this paper, we have also indicated the possibility that, in the final analysis, the *elaboration* via the scattering and isoscattering theories of the same *measured* data may lead to the same numerical results. This possibility should not be excluded due to the indicated preservation under isotopies of both Einsteinian and quantum axioms and, in case confirmed, would be quite valuable because it would confirm the broadening of their applicability under nonlinear, nonlocal and nonunitary internal effects.

However, even under the assumption that the data elaboration of *past* experiments are the same for the conventional; and the isotopic scattering theories, the latter is expected to admit the representation of events precluded to the former, such as the synthesis of neutrons from protons

and electrons as occurring in stars, or the synthesis of hadrons at large from lighter particles that, as we shall see in Paper IV of this series, can best be treated via a nonunitary-isounitary theory due to the need for a *negative binding energy* under which the Schrödinger equation no longer admits physically meaningful solutions [36].

Above all, the reader is suggested to keep in mind that the ultimate aim of all studies herein considered is the conception, quantitative treatment and experimental verification of much needed new clean energies, such as the novel *Intermediate Controlled Nuclear Fusions* (ICNF) [37], due to their strictly irreversible, as well as nonlinear, nonlocal and non-Hamiltonian character.

The proof of the convergence from primitive axioms without *ad hoc* manipulations, the comparison of the data elaboration of measured quantities via the scattering and isoscattering theory is done in subsequent papers. Similarly, the inclusion of antiparticles and the extension to irreversible scattering processes requires additional new mathematics (known as *isodual mathematics* and *Lie-admissible genomathematics,* respectively), thus requiring separate studies.

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NONUNITARY LIE-ISOTOPIC AND LIE-ADMISSIBLE SCATTERING THEORIES OF HADRONIC MECHANICS, II: Deformations-Isotopies of Lie's Theory, Special Relativity, and Mechanics

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Submission History

This paper and the remaining four of this series have been rejected by: G.t Hooft, editor of *Foundations of Physics;* L. Cifarelli, editor of *La Rivista Nuovo Cimento;* B. Nachtergaele, editor of *Journal of Mathematical Physics;* and E. Paal, editor of the *Journal of Generalized Lie Theories and Applications.* Additionally, Santilli stated *"I have not submitted the papers to APS and IoP journals due to their rejection of 'all' papers in the field since my last publications there in 1980* [PR D **22**, 892, 1980] *and 1983* [JoPG **18**, L141, 1983] *respectictively, all papers published in other refereed journals around the world.* In particular, t Hooft rejection was via a form letter without technical review; Cifarelli rejection included scientific wanderings plus the statement *"Moreover we do not wish receiving any further extension of your work.";* Nachtergaele blamed all JMP editors with the statement *"There was no support from the Editorial Board to publish the manuscripts."* (see the web site http://www.scientificethics.org/JMP.htm); and Paal's obstructed the very submission. The publisher submitted the papers to three outstanding physicists who released the following reviews: 1) *Following a lifelong research, Prof. Santilli has identified vast evidence on non-unitary effects in high energy inelastic scatterings requiring a re-inspection of 'experimental results' claimed via conventional unitary elaborations, with scientific, financial and ethical implications such that the suppression of the papers without due scientific process would constitute a crime against society;* 2) *Prof. Santilli's penetrating and impecccable analysis of scattering events establishes the need for the re-examination of basic paradigms due to experimentally established non-Lagrangian and non-Hamiltonian effects within the scattering region that must be debated in refereed publications for our community to verify minimal standards of ethic s and accountability;* 3) *Prof. Santilli's papers are: professionally written, technically outstanding, and potentially historical.* Therefore, without any consultation of the editors of the *Hadronic Journal,* the publisher elected to print the papers with the request to t Hooft, Cifarelli, Nachtergaele, Paal, APS-IoP editors and their friends that, in order to dispel increasing allegations of acting under orders by a scientific ring, they should publish their technical objections, if any possible, because science solely advances via publications and certainly not via equivocal 'reviews' voiced from opaque academic corridors. The series of five papers was finally accepted for publication by Kathmandu University, Nepal, following international review by qualified ethical scientists.The reasons for the political handling of these submissions are explained and denounced in the post by J. V. Kadeisvili Obscurantism at CERN on God's Particle? http://www.santilli-foundation.org/cern-obscurantism.php

This Submission History has been released by R. M. Santilli in his capacity as Founder and Honorary Editor in Chief of the Hadronic Journal without any consultation with any of the Editors.

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Abstract

In the preceding Paper I, we have presented a variety of aspects suggesting a reinspection of the elaboration of measured quantities (cross section, scattering angle, etc.) via the conventional unitary scattering theory due to possible non-Hamiltonian internal effects implying a nonunitary time evolution. We have then reviewed the inconsistency theorems for nonunitary theories on conventional spaces over conventional fields, outlined the foundations of the novel isomathematics permitting a resolution of said inconsistency theorems, and suggested an isounitary reformulation of nonunitary scattering theories. In this paper we outline the use of isomathematics to achieve of methods essential for a consistent treatment of nonunitary-isounitary theories for interior dynamical conditions, such as the deformations-isotopies of Lie's theory, special relativity and mechanics. The outline appears recommendable due to a variety of formulations existing in the literature often leading to misconceptions because of their inapplicability to scattering problems, or formulations prior to the resolution of the inconsistency theorems. Following this necessary background, the formulation of the isoscattering theory without divergencies $ab\ initio$ will be presented in Paper III, and comparative data elaborations via the conventional and the isotopic scattering theory will be initiated in Paper IV. ..

Key words: Lie algebnras, special relativity, Hamiltonian mechanics
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1. Deformations-Isotopies of Lie's Theory

1.1. Introduction. In the preceding paper [1], hereinafter referred to as Paper I, we have presented rather diversified conceptual, theoretical and experimental elements suggesting a reinspection of the validity of special relativity for interior dynamical problems at large, and the scattering region in particular.

R. M. Santilli had dedicated his lifetime of research to the construction of Lie-isotopic coverings of the Minkowskian geometry, the Lorentz-Poincaré symmetry and special relativity into forms more effective for interior conditions (see Refs. [3-16]). These covering formulations are, evidently, at the foundations of the Lie-isotopic scattering theory imder tyhe main assumptions indicated in Section 2 of Ref. [1], namely, the covering relativity and related covering symmetry are assumed for the interior of the scattering region, while all conventional formulations are recovered at distances bigger than $1\ fm$.

It is essential to outline in this paper the rather long scientific journey on the covering Lie-isotopic formulations for interior dynamical systems, due to the need of formulating them, specifically, for the scattering region. To begin our review, the central problem here referred to is the achievement of the universal invariance of locally varying speeds of light

$$C = \frac{c}{n(x, v, \xi, \omega, \psi, \partial\psi, ...)} \tag{1.1}$$

where c is the speed of light in vacuum, and $n(x, v, \xi, \omega, \psi, \partial\psi, ...)$ is the index of refraction generally dependent on local coordinates x, velocities v (e.g., of the medium with respect to the source), density of the medium ξ, frequency of light ω, wavefunction ψ, its derivative $\partial\psi$,and other variables. We assume the reader is aware from Paper I that local speed (1.1) is assumed in the isoscattering theory as applying also to photons, since they cannot be assumed, without due inspection, as propagating in vacuum when in the interior of the scattering region due to its hyperdense character.

In view of the primitive character of light for all of physics, the study of the isoscattering theory can be reduced to the study of photons propagating within a hyperdense scattering region composed of particles in

conditions of total mutual penetration. In the event the elaboration of measured quantities (cross section, scattering angles, etc.) via the isoscattering theory turns out to be entirely equivalent to the conventional elaboration, photons within the scattering regions would be confirmed as propagating in vacuum with consequential full validity of special relativity. By contrast, possible differences in the two data elaborations would establish that photons within the scattering region have local speed (1.1) with consequential need for a covering spacetime geometry, symmetry and relativity.

1.2. The Forgotten Lorentz Problem. Since the speed of light during pre-Einstein's time was considered to be a local quantity $C = c/n$, Lorentz [2] studied its invariance, as noted by Pauli in his celebrated book *Theory of Relativity,* but encountered major technical difficulties for the case of the index of refraction with an arbitrary functional dependence (rather than constant) and had to restrict his studies to the constant speed c, resulting in transformations that are now part of history.

To honor one of the founders of our physical knowledge, in these papers we shall refer to *the Lorentz problem* the achievement of the invariance of locally varying speeds of light with an arbitrary functional dependence of the index of refraction.

During the century following Lorentz studies, the invariance of locally varying speeds of light was forgotten due to the reduction of light to photons propagating in vacuum irrespective of whether in exterior or interior conditions, with consequential use of Lorentz's invariance for the constant speed c.

Via rigorously proved *No Reduction Theorems* indicated in Paper I, Santilli established the impossibility of a consistent reduction of interior to exterior conditions thus bringing back to life the Lorentz's problem as a beautiful problem *per se,* in view of its practical value irrespective of whether light is reducible or not to photons moving in vacuum, as well as for photons themselves.

The various conceptual, mathematical, theoretical and experimental needs to re-examine the scattering theory presented in Paper I, render Lorentz's problem one of the most important problems in contempo-

rary applied mathematics, whose solution can stimulate momentous advances in all quantitative sciences.

1.3. Insufficiencies of Lie's Theory. Santilli has dedicated his research life to the study of the Lorentz problem. The first contribution, as part of his Ph. D. Thesis in the mid 1960s, was to show that Lorentz's inability to achieve the desired invariance originated from insufficiencies of the background theory, *Lie's theory.* In fact, the applicability of said theory is notoriously restricted to linear, local and canonical systems at the classical level or unitary systems at the operator counterpart (where, in these papers, linearity is referred to the wavefunction, locality is referred to a finite set of isolated points, and canonicity or unitarity are referred to the respective time evolutions).

By contrast, the transition from the Minkowski metric characterizing the constant speed c to the deformed metric characterizing variable speed (1.1)

$$\eta = Diag.(1, 1, 1, -c^2) \ \rightarrow$$

$$\rightarrow \ \hat{\eta} = Diag.(1, 1, 1, -\frac{c^2}{n^2(x, v, \xi, \omega, \psi, \partial\psi, ...)}), \tag{1.2}$$

has been shown in Paper I as characterizing systems that are generally *nonlinear in the wavefuynction, nonlocal of integral character, and non-canonical or nonunitary in their time evolution,* It is then evident that Lie's theory. while so efective for the constant speed c, is generally *inapplicable* for the case of local speeds (1.1) (and certainly not "violated" because not conceived for the systems considered).

1.4. Santilli Lie-Admissible Covering of Lie's Theory. As part of his Ph. D. thesis, in order to broaden the representational capabilities of Lie's theory, Santilli proposed in 1967 [3] the first known *deformation of Lie algebras* in the physics literature with product

$$(A, B) = p \times A \times B - q \times B \times A, \tag{1.3}$$

where $p, q, p \pm q$ are non-null scalars (denoted λ and μ in Ref. [3]), A, B are matrices of the same dimension, and $A \times B$ is the conventional as-

sociative product according to the notations set forth in Paper I. Santilli called deformations (1.3) *mutations of Lie algebras* due to the evident loss of Lie's axioms, and proved that they characterize *Lie-admissible and Jordan-admissible algebras* according to Albert (in the sense that their attached antisymmetry and symmetric algebras are Lie and Jordan, respectively).

The proposal was intended to characterize the following *Lie-admissible generalization of Heisenberg's equations* for the dynamical evolution of a Hermitean operator A in the following infinitesimal and finite forms

$$i \times \frac{dA}{dt} = (A, H) = p \times A \times H - q \times H \times A, \tag{1.4a}$$

$$A(t) = e^{H \times q \times t \times i} \times A(0) \times e^{-i \times t \times q \times H}, \tag{1.4b}$$

By recalling that Lie algebras characterize closed-conservative systems reversible over time, proposal [3] essentially recommended the construction of a *Lie-admissible covering of Lie's theory* for the characterization of open, nonconservative and irreversible systems evidently in view of the non-null *time rate of variations of the energy* $i \times dH/dt = (H, H) \neq 0$.

In 1978, Santilli [4] proposed the *most general possible Lie-admissible and Jordan-admissible deformations-mutations of Lie algebras* with product

$$(A \hat{,} B) = A \times R \times B - B \times S \times A = A < B - B > A, \tag{1.5}$$

where $R, S, R \pm S$ are now fixed nonsingular operators with an arbitrary, nonlinear and nonlocal functional dependence on any needed quantity (including the wavefunction and its derivatives), which brackets resulted in characterize the *most general possible algebra* as known in mathematics (characterized by a bilinear composition law verifying the right and left distributive and scalar laws). Therefore, algebras with product (1.5) contain as particular cases associative, Lie, Jordan, supersymmetric, flexible and any other possible algebra.

Ref. [4] then presented the initiation of a joint *Lie-admissible and Jordan-admissible covering of Lie's theory* in its various branches, including the lifting of the universal enveloping algebra with generalized Poincaré-Birkhoff-Witt theorem, Lie algebras, Lie's (transformation) groups and the representation theory.

Product (1.5) was obtained by using the most general possible *nonunitary* transformation of product (1.3), and was suggested as the foundations of the following *Lie-admissible and Jordan-admissible deformations-mutations of Heisenberg's equations* proposed in the joint paper [5] with infinitesimal and finite forms

$$i \times \frac{dA}{dt} = (A\hat{,}H) = A \times R \times H - H \times S \times A = A < H - H > A, \quad (1.6a)$$

$$A(t) = e^{H \times S \times t \times i} \times A(0) \times e^{-i \times t \times R \times H}, \quad (1.6b)$$

$$R = S^{\dagger} \quad (1.6c)$$

The above equations were proposed as the foundations of *hadronic mechanics* for the representation of the most general possible open, nonconservative, irreversible and single-valued systems with potential interactions represented by the nonconserved Hamiltonian H, and contact nonpotential, nonlinear, nonlocal-integral and nonunitary interactions represented by the operators R, S.

Generalized dynamical equations (1.6) were originally formulated on conventional Hilbert spaces over conventional fields. Subsequent studies indicated that the equations verified the *Theorems of Catastrophic Mathematical and Physical Inconsistencies of Noncanonical and Nonunitary Theories* (see Refs. [6-12] of Paper I) because it is not preserving over time the basic units of measurements, the observability of physical quantities, the numerical predictions, etc.

The resolution of the above inconsistencies required decades of additional research. The first major advance occurred in 1993 with the discovery of the *genonumbers* and *genofields* [6], namely, fields with a fixed order of *all* multiplications to the right (representing motion forward in time) and an arbitrary right and left generalized unit called *genounit for the ordering to the right,*

$$n > m = n \times S \times m, \ \hat{I}^{>} = S^{-1}, \quad (1.7)$$

with the corresponding ordering of *all* multiplications to the left (representing motion backward in time) with related *genounit for the ordering to the left*

$$n < m = n \times R \times m, \ {}^{<}\hat{I} = R^{-1}, \quad (1.8)$$

where the word "genotopy" [4] was used in the Greek meaning of inducing new axioms.

In turn, the above genofields stimulated corresponding *two* genotopies, one to the right and, separately, one to the left, of functional analysis, metric spaces, geometries, enveloping associative algebras, etc. Despite all these efforts, the resolution of the inconsistency theorems remained elusive for years.

A breakthrough occurred in the mathematical memoir [7] of 1996 with the discovery of the new *genodifferential calculus to the right or to the left.* The first invariance over time of deformations-mutations of Lie algebras was proved in paper [8] of 1997. Final maturity in the axiomatic structure of Lie-admissible formulations was achieved in memoir [9] of 2006 that also presented the first known connection between mechanics and thermodynamics, by showing that the irreversibility of thermodynamical laws originates at the ultimate level of nature, in full confirmation of the No Reduction Theorems indicated earlier.

Readers should be aware that, in view of their only known axiomatically consistent characterization of irreversible processes (thus including energy releasing processes) in a way directly compatible with thermodynamics, Lie-admissible formulations have been the subject of rather vast studies since the time of Santilli's original proposal of 1978 [4], including mathematical, physical, chemical as well as industrial research (see monographs [10-27], references quoted therein and general biography in Volume [16a]).

We should mention that, twenty years following the origination of the parametric deformations [3] and ten years following the proposal of the operator deformations [4,5] (with related rather vast literature of the time including four monographs [10,11], five workshops on Lie-admissibility and an international conference [16a]), there was the appearance of a very large number of papers on parametric deformations of Lie algebras with the simpler product $A \times B - q \times B \times A$, generally without the quotation of their origination [3-5], as well as generally without the identification of their joint Lie-admissible and Jordan admissible character, despite their historical and technical values.

It is important for these papers to indicate that all the latter deforma-

tions have been proved to verify the theorems of catastrophic inconsistencies when formulated on conventional spaces over conventional fields (see Refs. [6-12] of Paper I). The words "deformations-isotopies" of the titles of the various sections of this paper stand to indicate that their field in applied mathematics is that nowadays vastly referred to as "deformations," although identically reformulated as "isotopies" to resolve said inconsistencies.

1.5. Santilli Lie-Isotopic Covering of Lie's Theory. These papers are intended for concrete applications to the elaboration of scattering data. As such, if initially presented with excessive mathematical complexities (as needed for the consistent treatment of irreversible scattering processes), these papers could be beyond the reach of most phenomenologists.

This is tsetting has suggested in Paper I the restriction of these initial studies to *reversible* scattering processes, and then the passage to the more complex irreversible events only subsequently. As an example, the restriction to reversible processes eliminates the need of the time ordering of all products, with consequential major simplification of the formalism.

Most importantly from the viewpoint of applied mathematics, *the restriction to reversible scattering processes permits the preservation of Lie's axioms, despite the admission of nonlinear, nonlocal and noncanonical or nonunitary effects.*

In fact, Santilli identified in the original proposal [4] of 1978, the following particularization of his Lie-admissible and Jordan-admissible product (1.5)

$$[A\hat{,}B] = A\hat{\times}B - B\hat{\times}A =$$

$$= A \times T(x, v, \xi, \omega, \psi, \partial\psi, ...) \times B - B \times T(x, v, \xi, \omega, \psi, \partial\psi, ...) \times A, \quad (1.9a)$$

$$R = S = T = T^{\dagger} > 0, \ \hat{I}(x, v, \xi, \omega, \psi, \partial\psi, ...) = 1/\hat{T}(x, v, \xi, \omega, \psi, \partial\psi, ...) > 0, \quad (1.9c)$$

where $\hat{I}(x, v, \xi, \omega, \psi, \partial\psi, ...)$ and $\hat{T}(x, v, \xi, \omega, \psi, \partial\psi, ...)$ are the *isounit* and the*isotopic element* at the foundation of the mathematics of Paper I, where one should note that quantities (1.9b) have the same functional dependence of local speed (1.1).

It is easy to verify that product (1.9) does indeed verify Lie's axioms. Consequently, the ensuing *deformations of Lie algebras* were called *isotopic* [4] by Santilli in their Greek meaning of preserving the original topology, a main characteristics that we have used in the very name of the isoscattering theory. In the same paper [4], Santilli then proposed a step by step isotopic generalization of Lie's theory that has remained structurally unchanged to this day (except for the subsequent reformulation on isospaces over isofields), and it is today known as the *Lie-Santilli isotheory* [18-27].

The main idea of said isotheory is that of preserving unchanged the generators of a given Lie symmetry and changing instead all their operations in an axiom-preserving way (as a condition to have an isotopy) [4]. The implementation of this idea require the lifting of the conventional associative product $A \times B$ into the axiom-reserving isoassociative form $A \times T \times B = A\hat{\times}B$ that, in turn, implies the lifting of the Lie product $[A.B]$ into the axiom-preserving form (1.9).

This seemingly elementary idea has important implications for the scattering theory. By recalling that the generator of a Lie symmetry represent conserved quantities, the preservation of the generators in the transition from the conventional to the isotopic scattering theory implies the preservation of all conventionally conserved quantities. However, the appearance of the isotopic element T in the product itself implies that said preservation occurs under nonlinear, nonlocal and noncanonical or nonunitary internal effects, thus warranting a reinspection of the data elaboration via the conventional linear, local and unitary scattering theory.

Since the covering isotheory is at the foundations of these papers, it appears recommendable to outline its main elements in a language accessible to phenomenologists and specialized to scattering problems, not only for notational scopes, but also to avoid possible insidious misrepresentations in the event of referral to a variety of seemingly different presentations existing in the literature. Also, most of the "objections" raised by colleagues in a shorter versions of these papers were essentially due to a lack of inspection of the Lie-Santilli isotheory in the disparate literature or, more insidiously, due to the inspection of presentations prior to

the achievement of invariance. Also, some of the results on Lie-isotopic studies are at times presented in the broader Lie-admissible context, as it is often the case of the original proposal [4].

As it was the case for other isotopies outlined in Paper I, the Lie-Santilli isotheory coincides with the conventional Lie theory at the abstract, realization-free level by conception and construction to such an extent that they can be presented at the pure mathematical level with the same symbols subjected to different realizations. However, such an abstract presentation would render quite difficult the practical applications of the isoscattering theory. Consequently, we shall outline below the specialization of the isotheory with emphasis on its applied version, namely, in its *projection* on conventional spaces over conventional fields.

Additionally, the reader should be aware that the original presentations verified the inconsistency theorems due to lack of invariance over time. In fact, the Lie-Santilli theory reached maturity only following the discovery of the *isonumbers* in 1993 [6] and of the *isodifferential calculus* in 1996 [7], discoveries that followed the otherwise excellent presentation by Tsagas and Sourlas [20] of 1993. Therefore, the outline below is based on all structural elements of the original proposal [4] formulated on isospaces over isofields [6] and via the isodifferential calculus [7].

UNIVERSAL ENVELOPING ISOASSOCIATIVE ALGEBRAS
Let $E = E(L)$ be the *universal enveloping associative algebra* of an N-dimensional Lie algebra L with ordered (Hermitean) generators $X_k,\ k = 1, 2, \ldots, N$, and attached antisymmetric algebra isomorphic to the Lie algebra, $[E(L)]^- \approx L$ over a field F (of characteristic zero), and let the infinite-dimensional basis $I, X_k, X_i \times X_j, i \leq j, \ldots$ of $E(L)$ be characterized by the *Poincaré-Birkhoff-Witt theorem.* We then have the following

THEOREM 1.5.1 [4]: (Poincaré-Birkhoff-Witt-Santilli theorem): The isocosets of the isounit and of the standard isomonomials

$$\hat{I},\ \ X_k,\ \ \hat{X}_i\hat{\times}\hat{X}_j,\ i \leq j,\ \ \hat{X}_i\hat{\times}\hat{X}_j\hat{\times}\hat{X}_k,\ i \leq j \leq k, \ldots, \tag{1.10}$$

form an (infinite dimensional) basis of the universal enveloping isoassociative algebra $\hat{E}(\hat{L})$ (also called isoenvelope for short) of a Lie-Santilli isoalgebra $\hat{L}$.

The first application of the above theorem, also formulated in Ref. [4] and then reexamined by various authors, is a rigorous characterization of the *isoexponentiation,* Eq. (3.4) of Paper I, i.e.,

$$\hat{e}^{\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X}} =$$

$$= \hat{I} + \hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X}\hat{/}\hat{1}! + (\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X})\hat{\times}(\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X})\hat{/}\hat{2}! + \ldots =$$

$$= \hat{I} \times (e^{i\times w\times T\times X}) = (e^{i\times w\times X\times T}) \times \hat{I}, \tag{1.11a}$$

$$\hat{i} = i \times \hat{I}, \hat{w} = w \times \hat{I} \in \hat{F}. \tag{1.11b}$$

where we continue to use the notation of Paper I according to which quantities with a "hat" are formulated on isospaces over isofields and those without are formulated on conventional spaces over conventional fields.

The nontriviality of the Lie-Santilli isotheory is illustrated by the emergence of the nonlinear, nonlocal and noncanonical or nonunitary isotopic element T directly in *the exponent,* thus ensuring the desired generalization.

LIE-SANTILLI ISOALGEBRAS.
As it is well known, Lie algebras are the antisymmetric algebras $L \approx [\xi(L)]^-$ attached to the universal enveloping algebras $\xi(L)$. This main characteristic is preserved although enlarged under isotopies as expressed by the following

THEOREM 1.5.2 [4] (Lie-Santilli Second theorem): The antisymmetric isoalgebras $\hat{L}$ attached to the isoenveloping algebras $\hat{E}(\hat{L})$ verify the iso-commutation rules

$$[\hat{X}_i\hat{,}\hat{X}_j] = \hat{X}_i\hat{\times}\hat{X}_j - \hat{X}_j\hat{\times}\hat{X}_i =$$

$$= X_i \times T(x, v, \xi, \omega, \psi, \partial\psi, ...) \times X_j - X_j \times T(x, v, \xi, \omega, \psi, \partial\psi, ...) \times X_i =$$

$$= \hat{C}^k_{ij}(x, v, \xi, \omega, \psi, \partial\psi, ...)\hat{\times}\hat{X}_k = C^k_{ij}(x, v, \xi, \omega, \psi, \partial\psi, ...) \times X_k, , \tag{1.12}$$

where the C's, called the "structure isofunctions" of $\hat{L}$, generally have an explicit dependence on local variables, and are restricted by the conditions (Lie-Santilli Third Theorem)

$$[X_i\hat{,}X_j] + [X_j\hat{,}X_i] = 0, \tag{1.13a}$$

$$[[X_i\hat{,}X_j]\hat{,}X_k] + [[X_j\hat{,}X_k]\hat{,}X_i] + [[X_k\hat{,}X_i]\hat{,}X_j] = 0. \qquad (1.13b)$$

It was stated in the original proposal [4] that *all isoalgebras $\hat{L}$ are isomorphic to the original algebra L for all positive-definite isotopic elements*. In other words, the isotopies *cannot* characterize any new Lie algebras algebra because all possible Lie algebras are known from Cartan classification. Therefore, Lie-Santilli isoalgebras merely provide *new nonlinear, nonlocal and noncanonical or nonunitary realizations of existing Lie algebras.*

LIE-SANTILLI ISOGROUPS.
Under certain integrability and smoothness conditions hereon assumed, Lie algebras L can be "exponentiated" to their corresponding Lie transformation groups G and, vice-versa, Lie transformation groups G admit corresponding Lie algebras L when computed in the neighborhood of the unit I.

These basic properties are preserved under isotopies although broadened to the most general possible, axiom-preserving nonlinear, nonlocal and noncanonical transformations groups according to the following:

THEOREM 1.5.3 [4] (Lie-Santilli iaogroups): The isogroup characterized by finite (integrated) form $\hat{G}$ of isocommutation rules (1.12) on an isospace $\hat{S}(\hat{x}, \hat{F})$ over an isofield $\hat{F}$ with common isounit $\hat{I} = 1/\hat{T} > 0$ is a group mapping each element $\hat{x} \in \hat{S}$ into a new element $\hat{x}' \in \hat{S}$ via the isotransformations

$$\hat{x}' = \hat{g}(\hat{w})\hat{\times}\hat{x}, \quad \hat{x}, \hat{x}' \in \hat{S}, \quad \hat{w} \in \hat{F}, \qquad (1.14)$$

with the following isomodular action to the right:
1) The map $\hat{g}\hat{\times}\hat{S}$ into $\hat{S}$ is isodifferentiable $\forall \hat{g} \in \hat{G}$;
2) $\hat{I}$ is the left and right unit

$$\hat{I}\hat{\times}\hat{g} = \hat{g}\hat{\times}\hat{I} \equiv \hat{g}, \quad \forall \hat{g} \in \hat{G}; \qquad (1.15)$$

3) the isomodular action is isoassociative, i.e.,

$$\hat{g}_1\hat{\times}(\hat{g}_2\hat{\times}\hat{x}) = (\hat{g}_1\hat{\times}\hat{g}_2)\hat{\times}\hat{x}, \quad \forall \hat{g}_1, \hat{g}_2 \in \hat{G}; \qquad (1.16)$$

4) in correspondence with every element $\hat{g}(\hat{w}) \in \hat{G}$ *there is the inverse element* $\hat{g}^{-\hat{I}} = \hat{g}(-\hat{w})$ *such that*

$$\hat{g}(\hat{0}) = \hat{g}(\hat{w})\hat{\times}\hat{g}(-\hat{w}) = \hat{I}; \tag{1.17}$$

5) the following composition laws are verified

$$\hat{g}(\hat{w})\hat{\times}\hat{g}(\hat{w}') = \hat{g}(\hat{w}')\hat{\times}\hat{g}(\hat{w}) = \hat{g}(\hat{w}+\hat{w}'), \forall \hat{g} \in \hat{G},\ \hat{w} \in \hat{F}; \tag{1.18}$$

with corresponding isomodular action to the left, and general expression

$$\hat{g}(\hat{w}) = \prod_k \hat{e}^{\hat{i}\hat{\times}\hat{w}_k\hat{X}_k}\hat{\times}\hat{g}(0)\hat{\times}\prod_k \hat{e}^{=\hat{i}\hat{\times}\hat{w}_k\hat{X}_k}, \tag{1.19}$$

Another important property is that conventional group composition laws admit a consistent isotopic lifting, resulting in the following

THEOREM 1.5.4 [4] (Baker-Campbell-Hausdorff-Santilli theorem):

$$(\hat{e}^{\hat{X}_1})\hat{\times}(\hat{e}^{\hat{X}_2}) = \hat{e}^{\hat{X}_3}, \tag{1.20a}$$

$$\hat{X}_3 = \hat{X}_1 + \hat{X}_2 + [\hat{X}_1\hat{,}\hat{X}_2]\hat{/}\hat{2} + [(\hat{X}_1 - \hat{X}_2)\hat{,}[\hat{X}_1\hat{,}\hat{X}_2]]\hat{/}\hat{12} + \dots\ . \tag{1.20b}$$

Let $\hat{G}_1$ and $\hat{G}_2$ be two isogroups with respective isounits $\hat{I}_1$ and $\hat{I}_2$. The *direct isoproduct* $\hat{G}_1\hat{\times}\hat{G}_2$ is the isogroup of all ordered pairs

$$(\hat{g}_1, \hat{g}_2), \quad \hat{g}_1 \in \hat{G}_1, \hat{g}_2 \in \hat{G}_2, \tag{1.21}$$

with isomultiplication

$$(\hat{g}_1, \hat{g}_2)\hat{\times}(\hat{g}'_1, \hat{g}'_2) = (\hat{g}_1\hat{\times}\hat{g}'_1, \hat{g}_2\hat{\times}\hat{g}'_2), \tag{1.22}$$

total isounit $(\hat{I}_1, \hat{I}_2)$ and inverse $(\hat{g}_1^{-\hat{I}_1}, \hat{g}_2^{-\hat{I}_2})$.

The following particular case is important for the isotopies of inhomogeneous groups. Let $\hat{G}$ be an isogroup with isounit $\hat{I}$ and $\hat{G}_{\hat{a}}$ the group of all its inner automorphisms. Let $\hat{G}^o_{\hat{a}}$ be a subgroup of $\hat{G}_{\hat{a}}$ with isounit

$\hat{I}^o$, and let $\Lambda(\hat{g})$ be the image of $\hat{g} \in \hat{G}$ under $\hat{G}_{\hat{a}}$. The *semidirect isoproduct* $\hat{G}\hat{\times}\hat{G}^o_{\hat{a}}$ is the isogroup of all ordered pairs $(\hat{g}, \hat{\Lambda})\hat{\times}(\hat{g}^o, \hat{\Lambda}^o)$ with total isounit

$$I_{tot} = \hat{I} \times \hat{I}^o. \tag{1.23}$$

The studies of the isotopies of the remaining aspects of the structure of Lie groups is then consequential. It is hoped the reader can see from the above elements that the entire conventional Lie theory does indeed admit a consistent and nontrivial lifting into the covering Lie-Santilli formulation.

Among a considerable number of mathematical papers on the Lie-Santilli isotheory listed in the Comprehensive Bibliography of Volume [16a], we quote in particular the readable review by J. V. Kadeisvili [28], an excellent presentation of the all fundamental isotopology by R. M. Falcon Ganfornina and J. Nunez Valdes [29], and the unification of all simple Lie algebras of a given dimension (excluding exceptional algebras) into one single Santilli isotope of the same dimension by Gr. T. Tsagas [30] (see also the review of the latter unification in Volume [16c]).

1.6. The Fundamental Theorem for Isosymmetries. The fundamental symmetries of the 20-th century physics characterize point-like abstractions of particles in vacuum under linear, local and potential interactions, and are given by the *Galilei symmetry* $G(3.1)$ for nonrelativistic treatment, the *Lorentz-Poincaré symmetry* for relativistic formulations, the $SU(3)$ symmetry for particle classifications, the gauge symmetry, and others.

A central objective of hadronic mechanics is the broadening of these fundamental symmetries to represent extended, nonspherical and deformable particles under linear and nonlinear, local and nonlocal and potential as well as nonpotential interactions *in such a way to preserve the original symmetries at the abstract level*.

This central objective is achieved by the following property first proved by Santilli in Ref. [13b]

THEOREM 1.6.1: Let G be an N-dimensional Lie symmetry of a K-dimen-

sional metric or pseudo-metric space $S(x,m,F)$ over a field F,

$$G: \quad x' = \Lambda(w) \times x, \quad y' = \Lambda(w) \times y, \quad x, y \in \hat{S}, \tag{1.24a}$$

$$(x'-y')^\dagger \times \Lambda^\dagger \times m \times \Lambda \times (x-y) \equiv (x-y)^\dagger \times m \times (x-y), \tag{1.24b}$$

$$\Lambda^\dagger(w) \times m \times \Lambda(w) \equiv m. \tag{1.24c}$$

Then, all infinitely possible isotopies $\hat{G}$ of G acting on the isospace $\hat{S}(\hat{x}, \hat{M}, \hat{F})$, $\hat{M} = \hat{m} \times \hat{I} = (\hat{T}_i^k \times m_{kj}) \times \hat{I}$ characterized by the same generators and parameters of G and the infinitely possible, common isounits $\hat{I} = 1/\hat{T} > 0$ leave invariant the isocomposition

$$\hat{G}: \quad x' = \hat{\Lambda}(w) \times x, \quad y' = \hat{\Lambda}(w) \times y, \quad x, y \in \hat{S}, \tag{1.25a}$$

$$(x'-y')^\dagger \times \hat{\Lambda}^\dagger \times \hat{m} \times \hat{\Lambda} \times (x-y) \equiv (x-y)^\dagger \times \hat{m} \times (x-y), \tag{1.25b}$$

$$\hat{\Lambda}^\dagger(\hat{w}) \times \hat{m} \times \hat{\Lambda}(\hat{w}) \equiv \hat{m}. \tag{1.25c}$$

and all infinitely possible so constructed isosymmetries $\hat{G}$ are locally isomorphic to the original symmetry G.

For a proof one may inspect Section 1.2 of Ref. [16b].

To achieve a technical understanding of the Lie-Santilli isotheory and of the isoscattering theory, the reader should note that, *while a given Lie symmetry G is unique as well known, there can be an infinite number of covering isosymmetries $\hat{G}$ with generally different explicit forms o the transformations due to the infinite number of possible isotopic elements.*

In fact, systems are characterized by the Hamiltonian H in the conventional scattering theory with trivial unit $I = Diag.(1, 1, ..., 1)$. In this case, changing the Hamiltonian implies the referral to a different system, but the symmetry transformations remain the same. In the isoscattering theory, systems are characterized by the Hamiltonian *H plus the isotopic element T*. In this case, changing the isotopic element implies the referral to a different systems as well as the characterization of generally different transformations due to the appearance of the isotopic element in the very structure of the isosymmetry.

Note also that all possible isosymmetries can be explicitly and uniquely constructed via the sole knowledge of the conventional symmetry and

the isotopic element. in fact, as implied by Theorem 1.5.5, the existence of the original symmetry plus the condition $\hat{I} > 0$ ensure verification of the integrability conditions for the existence of finite transformations, a property hereon tacitly implied.

1.7. Simple Construction of the Lie=Santilli Isotheory. A simple method has been identified in Refs. [13,16] for the construction of the Lie-Santilli isotheory, all its underlying isomathematics and all physical methods to be studied in the these papers. This method is important because it permits a simple implementation of scattering models into their isotopic form. The method consists in:

(i) Representing all conventional interactions with a Hamiltonian H and all non-Hamiltonian interactions and effects with the isounit $\hat{I}$;

(ii) Identifying the latter interactions with a nonunitary transform

$$U \times U^\dagger = \hat{I} \neq I \tag{1.26}$$

and

(iii) Subjecting the *totality* of conventional mathematical and physical quantities and all their operations to the above nonunitary transform, resulting in expressions of the type

$$I \to \hat{I} = U \times I \times U^\dagger = 1/\hat{T}, \tag{1.27a}$$

$$a \to \hat{a} = U \times a \times U^\dagger = a \times U \times U^\dagger = a \times \hat{I},\ a \in F, \tag{1.27b}$$

$$e^A \to U \times e^A \times U^\dagger = \hat{I} \times e^{\hat{T} \times \hat{A}} = (e^{\hat{A} \times \hat{T}}) \times \hat{I}, \tag{1.27d}$$

$$A \times B \to U \times (A \times B) \times U^\dagger =$$
$$= (U \times A \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times B \times U^\dagger) = \hat{A} \hat{\times} \hat{B}, \tag{1.27c}$$

$$[X_i, X_j] \to U \times [X_i X_j] \times U^\dagger =$$
$$= [\hat{X}_i \hat{,} \hat{X}_j] = U \times (C_{ij}^k \times X_k) \times U^\dagger = \hat{C}_{ij}^k \hat{\times} \hat{X}_k =$$
$$= C_{ij}^k \times \hat{X}_k, \tag{1.27e}$$

$$< \psi| \times |\psi > \to U \times < \psi| \times |\psi > \times U^\dagger =$$

$$=< \psi| \times U^\dagger \times (U \times U^\dagger)^{-1} \times U \times |\psi > \times (U \times U^\dagger) =$$
$$=< \hat{\psi}|\hat{\times}|\hat{\psi} > \times \hat{I}, \qquad (1.27f)$$
$$H \times |\psi > \to U \times (H \times |\psi >) = (U \times H \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times |\psi >) =$$
$$= \hat{H}\hat{\times}|\hat{\psi} >, etc. \qquad (1.27g)$$

The above simple rules permit the explicit construction of all needed *regular isotopies* as defined and illustrated in Section 1.9 (eigenvalkue preserving mapps), including: algebras, groups, symmetries, eigenvalues equations and all needed aspects [13]. It should be stressed that the above method is not applicable for the *irreguilar isotopies* as also defined and illustrated in Section 1.9 (eigenvalue mutating images) for which no map ois known at this writing.

Note finally that *serious inconsistencies emerge in the event even one single quantity or operation is not subjected to the above nonunitary map.* In the absence of comprehensive liftings, we would have a situation equivalent to the elaboration of quantum spectral data of the hydrogen atom with isomathematics, resulting in dramatic deviations from reality.

1.8. Invariance of the Lie-Santilli Isotheory. It is easy to see that the application of an additional nonunitary transform

$$W \times W^\dagger \neq I, \qquad (1.28)$$

to expressions (1.27) causes the *lack of invariance,* with consequential activation of the catastrophic inconsistency theorems reviewed in Paper I, such as the *change of the basic isounit*

$$\hat{I} \to \hat{I}' = W \times \hat{I} \times W^\dagger \neq \hat{I}, \qquad (1.29)'$$

that implies the loss of the represented system, let alone the lack of invariance of a physical theory over time, or the lack of invariance of an isosymmetry under its own action,

However, as indicated in Paper I, any given nonunitary transform can be identically rewritten in the isounitary form,

$$W \times W^\dagger = \hat{I}, \quad W = \hat{W} \times \hat{T}^{1/2}, \qquad (1.30a$$

$$W \times W^\dagger = \hat{W}\hat{\times}\hat{W}^\dagger = \hat{W}^\dagger\hat{\times}\hat{W} = \hat{I}, \tag{1.30b}$$

under which we have the invariance of the isounit and isoproduct [7]

$$\hat{I} \to \hat{I}' = \hat{W}\hat{\times}\hat{I}\hat{\times}\hat{W}^\dagger = \hat{I}, \tag{1.31a}$$

$$\hat{A}\hat{\times}\hat{B} \to \hat{W}\hat{\times}(\hat{A}\hat{\times}\hat{B})\hat{\times}\hat{W}^\dagger =$$
$$= (\hat{W} \times \hat{T} \times \hat{A} \times \hat{T} \times \hat{W}^\dagger) \times (\hat{T} \times \hat{W}^\dagger)^{-1} \times \hat{T} \times (\hat{W} \times$$
$$\times\hat{T})^{-1} \times (\hat{W} \times \hat{T} \times \hat{B} \times \hat{T} \times \hat{W}^\dagger) =$$
$$= \hat{A}' \times (\hat{W}^\dagger \times \hat{T} \times \hat{W})^{-1} \times \hat{B}' = \hat{A}' \times \hat{T} \times \hat{B}' = \hat{A}'\hat{\times}\hat{B}', \ etc. \tag{1.31b}$$

from which the invariance of the entire isotopic formalism follows.

Note that the invariance is ensured by the *numerically invariant values of the isounit and of the isotopic element under nonunitary-isounitary transforms,*

$$\hat{I} \to \hat{I}' \equiv \hat{I}, \tag{1.32a}$$

$$A\hat{\times}B \to A'\hat{\times}'B' \equiv A'\hat{\times}B', \tag{1.32b}$$

in a way fully equivalent to the invariance of Lie's theory and quantum mechanics, as expected to be necessarily the case due to the preservation of the abstract axioms under isotopies. The resolution of the inconsistencies for noninvariant theories is then consequential (see Paper I for details).

1.9. Regular and Irregular Pauli-Santilli Isomatrices. Due to the abstract identify of Lie and Lie-Santilli theories, as well as the simplicity of their interconnecting map of Section 1.7, it is at times believed that the isotopies are trivial. The best way to dispel this erroneous perception is via the isorepresentation theory for one of the central physical notions, that of spin.

Even though the isorepresentation theory is still vastly unexplored, the studies conducted until now have been sufficient to identify the existence of two classes, the *regular isorepresentations,* occurring under the preservation of the original structure constants, and the *irregular isorepresentations,* occurring under the alteration of the original structure constants.

The basic symmetries of the 20th century particle physics have been those of the *rotational symmetry* $SO(3)$ and the *spin symmetry* $SU(2)$. The corresponding isosymmetries $\hat{SO}(3)$ were studied by Santilli in the original proposal [5] of 1978 as well as in the two subsequent papers [31,32] of 1985. Isosymmetries $\hat{SU}(2)$ were first studied also by Santilli in paper [33] of 1993 and [34] of 1998 with the following main results:

CASE I: REGULAR PAULI-SANTILLI ISOMATRICES.
This is the case that, by definition, implies the *preservation of the conventional spin* $1/2$, although with new degrees of freedom nonexistent in the conventional notion of spin. The related regular two-dimensional irreducible isorepresentation of $\hat{SU}(2)$ are today known as *regular Pauli-Santilli isomatrices.*

This first notion of *hadronic spin,* that is, spin characterized by hadronic mechanics, is assumed for *low energy reversible scattering processes.* The assumption essentially implies that, as an example, an electron maintains its spin $1/2$ in the transition from motion in vacuum to motion within the scattering region, although in a generalized way identified below. As we shall see in Section 3, this assumption implies the preservation within the scattering region of the Fermi-Dirac statistics and Pauli's exclusion principle.

By remembering the lack of uniqueness of the isounits and related isotopic element, the simplest regular two-dimensional irreducible isorepresentations of $\hat{SU}(2)$ are characterized by the lifting of the two-dimensional complex-valued unitary space with metric $\delta = Diag.(1.1)$ into the isotopic image [33,34]

$$\hat{I} = Diag.((n_1^2, n_2^2),\ \hat{T} = Diag.(1/n_1^2, 1/n_2^2), \tag{1.33a}$$

$$\hat{\delta} = \hat{T} \times \delta = Diag.(1/n_1^2, 1/n_2^2), \tag{1.33b}$$

$$Det\ \hat{\delta} = (n_1 \times n_2)^{-2} = 1, \tag{1.33c}$$

with corresponding isounit and isotopic element

$$U\ times U^\dagger = \hat{I} = \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix},\ T = \begin{pmatrix} n_1^{-2} & 0 \\ 0 & n_2^{-2} \end{pmatrix}. \tag{1.34}$$

The related lifting of Pauli's matrices can then be easily constructed via the methods of Section 1.7 as follows

$$\sigma_k \ \rightarrow \ \hat{\sigma}_k = U \times \sigma_k \times U^{\dagger}, \tag{1.35a}$$

$$U = \begin{pmatrix} i \times n_1 & 0 \\ 0 & i \times n_2 \end{pmatrix}, U^{\dagger} = \begin{pmatrix} -i \times n_1 & 0 \\ 0 & -i \times n_2 \end{pmatrix}, [10pt] \tag{1.35b}$$

where the n's are well behaved nowhere null functions, resulting in the *regular Pauli-Santilli isomatrices* [*loc. cit*]

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & n_1^2 \\ n_2^2 & 0 \end{pmatrix}, \ \hat{\sigma}_2 = \begin{pmatrix} 0 & -i \times n_1^2 \\ i \times n_2^2 & 0 \end{pmatrix}, \ \hat{\sigma}_3 = \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}. \tag{1.36}$$

Another realization is given by *nondiagonal* nonunitary transforms [*loc. cit.*],

$$\begin{aligned} U &= \begin{pmatrix} 0 & n_1 \\ n_2 & 0 \end{pmatrix}, \quad U^{\dagger} = \begin{pmatrix} 0 & n_2 \\ n_1 & 0 \end{pmatrix}, \\ \hat{I} &= \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}, \quad \hat{T} = \begin{pmatrix} n_1^{-2} & 0 \\ 0 & n_2^{-2} \end{pmatrix}, \end{aligned} \tag{1.37}$$

with corresponding alternative version of the regular Pauli-Santilli isomatrices,

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & n_1 \times n_2 \\ n_1 \times n_2 & 0 \end{pmatrix}, \ \hat{\sigma}_2 = \begin{pmatrix} 0 & -i \times n_1 \times n_2 \\ i \times n_1 \times n_2 & 0 \end{pmatrix},$$

$$\hat{\sigma}_3 = \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}, \tag{1.38}$$

or by more general realizations with Hermitean nondiagonal isounits $\hat{I}$ [15b].

All Pauli-Santilli isomatrices of the above regular class verify the following *isocommutation rules and isoeigenvalue equations* on $\hat{\mathcal{H}}$ over $\hat{\mathcal{C}}$

$$[\hat{\sigma}_i, \hat{\sigma}_j] = \hat{\sigma}_i \times \hat{T} \times \hat{\sigma}_j - \hat{\sigma}_j \times \hat{T} \times \hat{\sigma}_i = 2 \times i \times \varepsilon_{ijk} \times \hat{\sigma}_k, \tag{1.39a}$$

$$\hat{\sigma}^{\hat{2}} \hat{\times} |\hat{\psi}\rangle =$$

$$(\hat{\sigma}_1 \times T \times \hat{\sigma}_1 + \hat{\sigma}_2 \times T \times \hat{\sigma}_2 + \hat{\sigma}_3 \times T \times \hat{\sigma}_3) \times T \times |\hat{\psi}\rangle = 3 \times |\hat{\psi}\rangle, \quad (1.39b)$$

$$\hat{\sigma}_3 \hat{\times} |\hat{\psi}\rangle = \hat{\sigma}_3 \times T \times |\hat{\psi}\rangle = \pm 1 \times |\hat{\psi}\rangle, \quad (5.39c)$$

thus preserving conventional structure constants and eigenvalues for spin $1/2$ under non-Hamiltonian/nonunitary interactions.

An interesting interpretation has been proposed in Ref. [31] for the case

$$n_1^2 = \lambda, \;\; n_2^2 = \lambda^{-1}, \quad (1.40)$$

according to which *the Pauli-Santilli isomatrices provide an explicit and concrete realization of a kind of hidden variables,* in the sense that the variable λ is indeed hidden in the axioms of the $SU(2)$ symmetry, with the understanding that we are not referring to the traditional interpretation of hidden variables, such as the historical one by Bohm. Note that this new degree of freedom is absent in the conventional Lie theory and can be solely identified via the Lie-Santilli isotheory.

Irrespective of the type of hidden variable we are here referring to, the Pauli-Santilli isomatrices with characteristic quantity (1.40) have caused a reinspection of Bell's inequalities, local realism and all that due to the strictly unitary structure of the latter compared to the nonunitary character of the former. We regret being unable to outline these intriguing new vistas, and refer the interested reader to paper [34].

CASE II: IRREGULAR PAULI-SANTILLI ISOMATRICES.
As it is well known by experts in quantum mechanics, action-at-a-distance, potential interactions leave invariant the *intrinsic* characteristics of particles, such as spin. By comparison, as well known by experts in hadronic mechanics, contact non-Hamiltonian interactions generally cause alterations, called mutations, of all *intrinsic* characteristics of particles, including spin.

According to the Lie-Santilli isotheory, the mutations for spin $1/2$ are characterized by the *irregular two-dimensional irreducible representations* of $\hat{SU}(2)$ known as the *irregular Pauli-Santilli isomatrices* that, by definition, do not preserve the spin $1/2$ and, consequently, cannot be constructed via nonunitary transformations of conventional representations.

This case is assumed for the representation of particles at high energy originally having spin $1/2$ when penetrating within hyperdense hadronic

media, whether existing in the core of stars or inside very high energy scattering region. The main argument is that the belief that an electron preserves its spin $1/2$ when in the core of a star does not appear to be plausible on various grounds, such as the loss of conventional quantized states within hyperdense media, the impossibility under the same conditions to possess a conserved angular momentum, and other reasons [13,16].

One illustrative example of irregular Pauli-Santilli isomatrices is given by [33.34]

$$\tilde{\sigma}_1 = \begin{pmatrix} 0 & n_1^2 \\ n_2^2 & 0 \end{pmatrix}, \ \ \tilde{\sigma}_2 = \begin{pmatrix} 0 & -i \times n_1^2 \\ i \times n_2^2 & 0 \end{pmatrix}, \ \ \tilde{\sigma}_3 = \begin{pmatrix} w \times n_1^2 & 0 \\ 0 & w \times n_2^2 \end{pmatrix}. \tag{1.41}$$

where w is the *mutation parameter,* with isocommutation rules

$$[\tilde{\sigma}_1,\tilde{\sigma}_2] = i \times w^{-1} \times \tilde{\sigma}_3, \ \ [\tilde{\sigma}_2,\tilde{\sigma}_3] = i \times w \times \tilde{\sigma}_1, \ \ [\tilde{\sigma}_3,\tilde{\sigma}_2] = i \times w \times \tilde{\sigma}_1, \tag{1.42}$$

and isoeigenvalues

$$\tilde{\sigma}^{\hat{2}} \hat{\times} |\hat{\psi}\rangle =$$

$$(\tilde{\sigma}_1 \times T \times \tilde{\sigma}_1 + \tilde{\sigma}_2 \times T \times \tilde{\sigma}_2 + \tilde{\sigma}_3 \times T \times \tilde{\sigma}_3) \times T \times |\hat{\psi}\rangle = (2 + w^2) \times |\hat{\psi}\rangle, \tag{1.43a}$$

$$\tilde{\sigma}_3 \hat{\times} |\hat{\psi}\rangle = \tilde{\sigma}_3 \times T \times |\hat{\psi}\rangle = \pm w \times |\hat{\psi}\rangle, \ \ w \neq 1, \tag{1.43b}$$

Additional examples of irregular Pauli-Santilli isomatrices can be found in Refs. [13,16].

The assumption of a mutated spin in hyperdense interior conditions evidently implies the inapplicability (rather than the violation) of the Fermi-Dirac statistics, Pauli's exclusion principle and other quantum mechanical laws, with the understanding that, by central assumption of Paper I, the scattering region as a whole must have conventional total quantum values because it is inspected from exterior conditions. Therefore, we are here referring to possible internal exchanges of angular momentum always in such a way to cancel out and yield total conventional values,

It should be indicated that we are here stressing the need to establish our knowledge in interior conditions via experiments rather than unverified assumptions, for which reason the isoscattering theory is proposed

in the first place. The need to *test Pauli's exclusion principle under "external" strong interactions* was stressed since the title of paper [5] of 1978 and, after some 32 years, that call remains more valid than ever.

2.Deformations-Isotopies of Special Relativity.

2.1. Introduction. Following decades of research on the deformations-isotopies of Lie's theory, Santilli was finally in a position to construct the deformations-isotopies of all main aspects of the conventional *Lorentz-Poincaré (LP) symmetry,* including the isotopies of: the rotational symmetry [4,31,32]; the $SU(2)$-spin symmetry [33,34]; the Lorentz symmetry at the classical [35] and operator [36] levels; the Poincaré symmetry [37]; the spinorial covering of the Poincaré symmetry [38,39]; and the isotopies of the Minkowskian geometry 40]. The new symmetry is today known as the *Lorentz-Poincaré-Santilli isosymmetry,* or *LPS isosymmetry,* for short [18-27].

Following all the above preparatory research, Santilli was finally in a position to study the deformations-isotopies of special relativity into a form providing the invariant (rather than covariant) characterization of interior dynamical problems at large, including the interior of the scattering region, the locally varying speed of light or photons (1.1).

The difficulties inherent in the realization of this objective were compounded by Santilli's specific intent of *honoring Albert Einstein via the preservation of his axioms for interior dynamical problems, and the mere presentation of broader realizations,* so as to avoid the abuse of Einstein's name via the application of his axioms under conditions never intended for and never directly tested.

The above objective was achieved thanks to the universal LPS isosymmetry, as well as its local isomorphism to the conventional LP symmetry, resulting in the axiom-preserving deformations-isotopies of special relativity first presented in Refs. [35,36] of 1983 at the classical and operator levels, respectively, and then studied in a variety of subsequent works (see monographs [12] of 1991 for the first systematic treatment and subsequent presentations in monographs [13] of 1995 and [16] of 2008 with literature quoted therein), resulting in a covering relativity today known as *Santilli isorelativity* [18-27].

It should be indicated that numerous "deformations" of the Minkowski space, the Lorentz-Poincaré symmetry and special relativity exist in the literature. However, to our best knowledge, all of them appeared a decade following the original proposal [35] by generally adopting the same symbols and main terminology, often without the quotation of the originating works [35]. Numerous other attempts at generalizing special relativity exist in the literature of the past century, although they do not possess a universal symmetry, thus lacking uniqueness in their derivation.

All these studies are noncanonical at the classical level and nonunitary at the operator level as an evidently necessary condition for novelty, and are formulated on conventional spaces over conventional fields. As such, all these studies directly verify the *Theorems of Catastrophic Inconsistencies of Noncanonical and Nonunitary Theories*, Refs. [6.-12] of Paper I.

We regret to be unable to review these studies to prevent an excessive length, as well as risk partial, thus discriminatory listings. Nevertheless, it is hope that interested colleagues may inspect preceding broadening of special relativity because it is the hope of all theories, including those here proposed, to contain at best a grain of truth, and comparative analyses of different approaches are always scientifically valuable.

The isoscattering theory is based on Santilli deformations-isotopies of the Minkowskian geometry, the Lorentz-Poincaré symmetry and special relativity because said isotopies:

1) are *directly universal*, that is, admitting as particular cases of all possible $(3+1)$-dimensional generalizations of the Minkowskian spacetime (universality) directly in the frame of the experimenter without any transformation to hypothetical reference frames (direct universality), and have been proved to include as particular cases all other possible deformations via different expansions in terms of different parameters and with different truncations [41-43], thus reducing a variety of possibilities to a primitive isosymmetry [41-43];

2) have resolved said inconsistency theorems, thus being consistently applicable to actual measurements [13,16]; and

3) have significant experimental verifications in classical physics, particle physics, nuclear physics, superconductivity, chemistry, biology, as-

trophysics and cosmology (see Ref. [16d], Chapter 5 of Ref. [27] and paper [47]).

As it was the case for the Lie-Santilli isotheory, the objections received by the authors on an earlier and shorter version of these papers on the deformations-isotopies of special relativity, were primarily due to the inspection of inappropriate literature or to inconsistent presentations because not formulated on isospaces over isofields. Consequently, it appears recommendable to review the foundational elements of the field specialized to the scattering problem.

2.2. Deformation-Isotopies of the Minkowski Spacetime. As it is well known, the carrier space of relativistic scattering theory is the familiar Minkowski space $M(x, \eta, \mathcal{R})$, where we assume in these papers $x = (x^\mu), \mu = 1, 2, 3, 4, x^k = r^k, k = 1, 2, 3, x^4 = t$, and $\eta = Diag.(1, 1, 1, -c^2)$. Such a space is crucially dependent on the assumed basis unit, that of the Lorentz symmetry $I = Diag.(1, 1, 1, 1)$.

As customary in *relativistic hadronic mechanics* [13,16], the Minkowski spacetime with related Lorentz-Poincaré symmetry and special relativity are assumed as being *exact for the conditions clearly indicated by Einstein, i.e., for point-like particles and electromagnetic waves propagating in vacuum conceived as empty space,* under which conditions we have the constancy of the speed of light c for all possible inertial systems. Therefore, special relativity is assumed as being exact everywhere in the *exterior* of the scattering region.

For the interior of the scattering region, as indicated in Section 2 of Paper I and studied in more details in Ref. [47], there are no possible inertial reference frames and we solely have the privileged frame at rest with the scattering region itself. Additionally, according to incontrovertible experimental evidence, the high energy scattering region is not an empty sphere with point-like particles in its interior, as requested by the mathematical structure of quantum mechanics. Instead, the scattering region is a hyperdense medium characterized by the mutual penetration of the wavepackets of scattering particles irrespective of whether their charge distribution is extended or point-like.

The above and other aspects imply that *in the interior of the scatter-*

ing region the speed of light in general and that of photons in particular is assumed as being a local variable $C = c/n(x, v, \xi, \omega, \psi, \partial\psi, ...)$ according to Eq. (1.1). Most importantly, photons cannot be assumed as propagating in vacuum when in the interior of the scattering region due to its hyperdense character, as indicated in Section 1.1.

The locally varying character of the speed of light is geometrically represented via the assumption that *physical media alter the geometry of spacetime.* This assumption is necessary for any geometric representation of the variation of the speed of light, $c \to C = c/n(x, v, \xi, \omega, \psi, \partial\psi, ...)$. Equivalently, we can say that no variation of the speed of light is possible without a corresponding alteration of spacetime.

Following decades of studies, Santilli [35.36] proposed in 1983 the representation of the alteration of spacetime via the (axiom-preserving) deformations-isotopies of the Minkowski spacetime, today called *Minkowski-Santilli isospacetimes,* or *isospacetimes* for short, generally indicated with the symbols $\hat{M}(\hat{x}, \hat{\eta}, \hat{\mathcal{R}})$, and characterized by:

A) The *isounit* and related *isotopic element* usually assumed (from their positive-definiteness) to have the diagonal form (see Ref. 13b] for off-diagonal realizations)

$$\hat{I} = Diag(1/b_1^2, 1/b_2^2, 1/b_3^2, 1/b_4^2) = Diag.(n_1^2, n_2^2, n_3^2, n_4^2), \tag{2.1a}$$

$$\hat{T} = Diag(b_1^2, b_2^2, b_3^2, b_4^2) = Diag.(1/n_1^2, 1/n_2^2, 1/n_3^2, 1/n_4^2), \tag{2.1b}$$

B) The *isometric*

$$\Xi = \hat{T} \times \hat{m} = (T \times \eta) \times \hat{I} = \hat{\eta} \times \hat{I} =$$

$$= [Diag(b_1^2, b_2, b_3^2, -c^2 \times b_4^2) \times \hat{I} = Diag.(1/n_1^2, 1/n_2^2, 1/n_3^2, -c^2/n_4^2) \times \hat{I}, \tag{2.2}$$

C) The *isoinvariant*

$$\hat{x}^{\hat{2}} = \hat{x}^\mu \hat{\times} \hat{\Xi}_{\mu\nu} \hat{\times} \hat{x}^\nu = (x^\mu \times \hat{\eta}_{\mu\nu} \times x^\nu) \times \hat{I} =$$

$$= x^1 \times b_1^2 \times x^1 + x^2 \times b_2^2 \times x^2 + x^3 \times b_3^2 \times x^3 - t \times c \times b_4^2 \times t \times c) =$$

$$= (\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - t^2 \times \frac{c^2}{n_4^2}) \times \hat{I}, \tag{2.3}$$

where one should note that $\hat{\Xi}$ is an isomatrix (because its elements are isonumbers), while $\hat{\eta}$ is an ordinary matrix.

The quantities $b_\mu = 1/n_\mu$ are called the *characteristic quantities* of the considered scattering region that can be averaged to constants, as we shall see in Paper III. A rather frequent erroneous perception is that the quantities $b_\mu = 1/n_\mu$ are arbitrary parameters, while in reality they represent *measurable quantities.*

In fact, the space isounits $\hat{I}_k^k = n_k^2$ characterize the actual *size* (thus being of the order of $10^{-13}cm$) and *shape* (say, a spheroid ellipsoid)) of the scattering region that are indeed measurable. By contrast, "parameters" can assume arbitrary values thus generally having no connection with the actual size and shape of the physical region considered. In the elaboration of the isoscattering theory, the space isounits $\hat{I}_k^k = n_k^2$ are normalized to the perfect sphere of radius $\hat{I}_k^k = 1fm$.

Similarly, the time isounit $\hat{I}_4^4 = n_4^2$ is a direct representation of the index of refraction that, as such, cannot possibly be a "parameter." More specifically, the time isounit provides a geometrization of the *density* of the scattering region (defined as the ratio between its energy and volume). In the isoscattering theory, the time isounit is normalized to the value for the vacuum $\hat{I}_4^4 = n_4^2 = 1$ for which $C = c/n = c$.

Finally, the characteristic quantities allow a representation of the *inhomogeneity* of physical media, e.g., via a dependence of the isounits on the local coordinates), as well as of their *anisotropy* (e.g., via different values between the space and time components of the isounit). Therefore, *the isoscattering theory allows, for the first time, a direct representation (i.e., a representation via the isometric) of the size, shape, density, inhomogeneity and anisotropy of the scattering region.*

The distinction between "parameters" and "characteristic quantities" is best illustrated by the Bose-Einstein correlation. Its treatment via relativistic quantum mechanics requires *four* arbitrary parameter of unknown origin, called "chaoticity parameters," that are fitted from the experimental data. By contrast, the representation of the same experimental data via relativistic hadronic mechanics yields space characteristic quantities providing a numerical representation of the actual shape of the fireball (a very elongated ellipsoid), while the forth component provides a nu-

merical, representation of the density of the fireball in a way consistent with other experiments (see Ref. [16d], Chapter 5 of Ref. [27] and papers quoted therein).

The *Minkowski-Santilli isogeometry* has been worked out in detail in Ref. [40]. Its main implication is that the resulting geometrization of the interior scattering region includes in a unified form *all possible geometries in* $(3+1)$*-dimensions,* thus including the Riemannian, Finslerian and other geometries that are merely differentiated by the assumed isounit, although all formulated via the abstract axioms of the *Minkowski* space. Such a broadening is necessary for any realistic representation of scattering regions, e.g., because interior dynamical problems generally require metrics with an explicit dependence on velocities and other variables, thus rendering the sole Riemannian description excessively restrictive.

2.3. Deformations-Isotopies of the Lorentz-Poincaré Symmetry. Following, and only following the prior construction of the deformations-isotopies of Lie's theory outlined in Section 1.5, Santilli constructed systematic, step-by-step deformations-isotopies of all spacetime symmetries [12,31-40], including the isotopies of the Galilei symmetry and of the Lorentz-Poincaré symmetry.

Evidently, we cannot possibly review here these studies in details. However, to render this presentation minimally selfsufficient, we outline the rudiments of the *regular Lorentz-Poincaré-Santilli (LPS) isosymmetry* $\hat{P}(3.1)$ [37] specialized to the scattering problem, and leave to the interested reader the study of the nonrelativistic-Galilean counterpart from monographs [12].

By using the second theorem, Eq. (1.12), the *regular LPS isoalgebra* is characterized by the conventional generators and the isocommutation rules [37]

$$[J_{\mu\nu},\hat{,}J_{\alpha\beta}] = i \times (\hat{\eta}_{\nu\alpha} \times J_{\beta\mu} - \hat{\eta}_{\mu\alpha} \times J_{\beta\nu} - \hat{\eta}_{\nu\beta} \times J_{\alpha\mu} + \hat{\eta}_{\mu\beta} \times J_{\alpha\nu}), \quad (2,4a)$$

$$[J_{\mu\nu},\hat{,}P_{\alpha}] = i \times (\hat{\eta}_{\mu\alpha} \times P_{\nu} - \hat{\eta}_{\nu\alpha} \times P_{\mu}), \quad (2,4b)$$

$$[P_{\mu},\hat{,}P_{\nu}] = 0, \quad (2,4c)$$

The *iso-Casimir invariants* of $\hat{P}(3.1)$ are given by [37]

$$\hat{C}_1 = \hat{I}(x, ...), \tag{2.5a}$$

$$\hat{C}_2 = P^{\hat{2}} = P_\mu \hat{\times} P^\mu = P^\mu \times \hat{\eta}_{\mu\nu} \times P^\nu = \\ = P_k \times g_{kk} \times P_k - p_4 \times g_{44} \times P_4, \tag{2,5b}$$

$$\hat{C}_2 3 = W^{\hat{2}} = W_\mu \hat{\times} W^\mu, \;\; W_\mu = \hat{\epsilon}_{\mu\alpha\beta\rho} \hat{\times} J^{\alpha\beta} \hat{\times} P^\rho, \tag{2.5c}$$

and they are at the foundation of classical and operator *isorelativistic kinematics* [13,16].

Since $\hat{I} > 0$, it is easy to prove that *the LPS isosymmetry is isomorphic to the conventional symmetry.* It then follows that *the isotopies increase dramatically the arena of applicability of the Lorentz-Poincaré symmetry, from the sole Minkowskian spacetime to all infinitely possible isospacetimes* (2.3).

By using Theorem 1.5.3 on the isogroup, the main components of the *regular LPS isotransformations* can be presented as follows in their projection on conventional spacetime:

(1) **Regular isorotations** $\hat{SO}(3)$, first presented in Ref. [31,32], here expressed in the (1,2)-plane (see monograph [13b] for the general case)

$$x^{1'} = x^1 \times \cos[\theta \times (n_1 \times n_2)^{-1}] - x^2 \times \frac{n_1^2}{n_2^2} \times \times \sin[\theta \times (n_1 \times n_2)^{-1}], \tag{2.6a}$$

$$x^{2'} = x^1 \times \frac{n_2^2}{n_1^2} \times \sin[\theta \times (n_1 \times n_2)^{-1}] + x^2 \times \cos[\theta \times (n_1 \times n_2)^{-1}], \tag{2.6b}$$

The isotopies of the $SU(2)$ symmetry were outlined in Section 1.9.

It was popularly believed in the 20th century that the $SO(3)$ symmetry is broken for the ellipsoidic deformations of the sphere. However, it is easy to prove that $\hat{SO}(3)$ is isomorphic to $SO(3)$, e.g., because, in the transition from a Lie symmetry to its isotopic covering, the original generators, parameters and structure constants remain unchanged.

Conceptually, this is due to the fact that ellipsoid deformations of the semiaxes of the perfect sphere are compensated on isospaces over isofields by the *inverse* deformation of the related unit

$$Radius \quad 1_k \to 1/n_k^2, \;\; Unit \quad 1_k \to n_k^2. \tag{2.7}$$

resulting in the reconstruction of the perfect sphere on isospace called the *isosphere*,

$$\hat{r}^{\hat{2}} = \hat{r}_1^{\hat{2}} + \hat{r}_2^{\hat{2}} + \hat{r}_3^{\hat{2}}. \tag{2.8}$$

with consequential reconstruction of the exact rotational symmetry.

Alternatively, we can say that the reconstruction of the exact rotational symmetry is due to the structure of the basic invariant given by

$$L^2 = (length)^2 \times (unit)^2 \tag{2,9}$$

Consequently, a change of lengths joint with the inverse change of units leaves the invariant unchanged.

Similarly we have the reconstruction of the exact isospin symmetry in nuclear physics under electromagnetic interactions via the simple mechanism of embedding all symmetry breaking terms in the isounit [34].

(2) **Regular Lorentz-Santilli isotransforms** $\hat{SO}(3.1)$, first identified in Ref. [35] here presented for simplicity in the (3-4)-plane (see monograph [13b] for the general case)

$$x^{1'} = x^1, \;\; x^{2'} = x^2, \tag{2.10a}$$

$$x^{3'} = \hat{\gamma} \times (x^3 - \hat{\beta} \times \frac{n_3}{n_4} \times x^4), \tag{2.10b}$$

$$x^{4'} = \hat{\gamma} \times (x^4 - \hat{\beta} \times \frac{n_4}{n_3} \times x^3), \tag{4.10c}$$

where

$$\hat{\beta}^2 = \frac{v_3^2/n_3^2}{c_o^2/n_4^2}, \;\; \hat{\gamma} = \frac{1}{\sqrt{1 - \hat{\beta}^2}}. \tag{2.11}$$

The isotopies of the spinorial covering of $SO(3.1)$ were studied or the first time in Ref. [38]. (see also monograph [13b]).

Again, it was popularly believed in the 20th century that the Lorentz symmetry is broken for deformations of the light cone. By contrast, Ref. [37] proved that the Lorentz symmetry does remain exact under the deformations of the light cone, provided it is treated with the appropriate mathematics. This result was achieved via the proof of the local isomorphism between $\hat{SO}(3.1)$ and $SO(3.1)$.

Conceptually, this is due to the reconstruction of the exact light cone on isospace over isofields called the *light isocone*. In fact, jointly with the deformation of the light cone

$$x^2 = x_3^2 - t^2 \times c^2 = 0 \rightarrow \frac{x_3^2}{n_3^2} - t^2 \times \frac{c^2}{n^4} = 0, \tag{2.12}$$

we have the corresponding inverse deformations of the units, thus reconstructing the original light cone on isospaces over isofields,

$$\hat{x}^{\hat{2}} = \hat{x}_3^{\hat{2}} - \hat{t}^{\hat{2}} \times \hat{c}^{\hat{2}} = 0. \tag{2.13}$$

The reader should be aware that the above reconstruction includes the preservation on isospace over isofields of the original characteristic angle of the conventional light cone. Consequently, *the maximal causal speed on isospace over isofields is the conventional speed of light in vacuum c*.

The understanding of the isoscattering theory requires the knowledge that *in the transition from the exterior to the interior region (as depicted in Figure 2 of Paper I), the speed of light and related light cone remain unchanged, and only their realizations change.*

(3) **Regular isotranslations** $\hat{\mathcal{T}}(4)$, first studied in ref. [37] (see monograph [13b] for the general case). can be expressed with the following lifting of the conventional translations $x^{\mu'} = x^\mu + a^\mu, \mu = 1, 2, 3, 4,$ and a^μ constants,

$$x^{\mu'} = x^\mu + A^\mu(a, \ldots), \tag{2.14}$$

where

$$A^\mu = a^\mu(n_\mu^{-2} + a^\alpha \times [n_\mu^{-2}\hat{,}P_\alpha]/1! + \ldots), \tag{2.15}$$

and there is no summation on the μ indices.

Readers should note the highly nonlinear character of the above translations. Nevertheless, the components of the linear momentum isocommute,

$$[P\mu\hat{,}P\nu] = P\mu \times T \times P\nu - P\nu \times T \times P\mu = 0, \ \mu, \nu = 1, 2, 3, 4. \tag{2.16}$$

while the generators of nonlinear transformations (2.14)-(2,15) do not commute in conventional space,

$$[P\mu, P\nu] = P\mu \times P\nu - P\nu \times P\mu \neq 0, \ \mu, \nu = 1, 2, 3, 4. \tag{2.17}$$

As we shall indicate in the next section, the above occurrence illustrates the capability by isorelativity of turning *curved* spaces into an equivalent *isoflat space* [40] with nontrivial implications, such as a consistent operator formulation of gravity, new grand unification, new interior gravitational models and other advances.

Recall that the conventional scattering theory is based on the conventional Minkowski space and, therefore, has no gravitational contribution of any type. By contrast, the above features imply that *the isoscattering region is isoflat and, therefore, with a primary gravitational, Finslerian and other contributions.*

4) **Regular isodilations** $\hat{\mathcal{T}}(1)$, first identified in Ref. [37] (see, again, monograph [13b] for the general case)

$$\hat{\eta} \to \hat{\eta}' = w^{-1} \times \hat{\eta},\ \hat{I} \to \hat{I}' = w \times \hat{I}, \tag{2.18a}$$

$$(x^\mu \times \hat{\eta}_{\mu\nu} \times x^\nu) \times \hat{I} \equiv [x^\mu \times (w^{-1}\hat{\eta}_{\mu\nu}) \times x^\nu) \times (w \times \hat{I}) =$$
$$= (x^\mu \times \hat{\eta}'_{\mu\nu} \times x^\nu) \times \hat{I}',\ w \in \mathcal{R}. \tag{2.18b}$$

As one can see, *the LPS isosymmetry is eleven dimensional,* the 11-th dimensionality being given by the new *invariance* under isodilations similar to that of the Hilbert space, Eq. (3.19) of Paper I. Contrary to popular beliefs in the 20th century, *the conventional Lorentz-Poincaré symmetry is also eleven dimensional,* trivially, because isoinvariance (2.18) also applies to the standard case.

Readers should keep in mind the transition from the conventional dilations, often requiring an enlargement of the spacetime dimensions, to their invariant formulation under isotopies within the conventional (3.1)-dimensions.

Predictably, the discovery of a new symmetry for the conventional spacetime has momentous implications. In fact, isosymmetry (2.18) was instrumental for the first known axiomatically consistent operator formulation of gravity [44], grand unification of electroweak and gravitational interactions [45,46], and other basic advances.

The reason that the eleventh dimensionality was not discovered until Ref. [37] of 1993 should not be surprising since the new isosymmetry required the prior discovery of *new numbers,* the isonumbers with arbitrary units [6].

The resulting eleven-dimensional regular LPS isosymmetry can be written

$$\hat{P}(3.1) = [\hat{SO}(3.1)\hat{\times}\hat{\mathcal{T}}(3.1)] \times \hat{\mathcal{C}}(1), \tag{2.19}$$

where $\hat{\times}$ is the direct isoproduct and $\times$ is the Kronecker product.

Readers should finally keep in mind the "direct universality" of the LPS symmetry indicated in Section 2.1 since said symmetry provides the universal *invariance* (rather than covariance) of all infinitely possible line elements (2.3) that include as particular cases the Riemannian, Finslerian or other line elements in $(3+1)$-dimensions [41-43].

the isotopies of the spinorial covering of the LP symmetry will be indicated in Section 3 jointly with the isotopies of Dirac's equation.

2.4. Deformations-Isotopies of Special Relativity. As indicated in Section 2.1, a central objective of the studies here considered is the preservation of Einstein axioms for interior conditions and the enlargement of their applicability via broader realizations. This objective was achieved first in paper [35] of 1983. Comprehensive elaborations were presented in monographs [12] of 1991 that include the isotopies of Galilei relativity we cannot possibly review here. Additional presentations are available in monographs [13] of 1995 and [16] of 2008.

As a result of these studies, *special relativity and its covering isorelativity can be presented at the abstract mathematical level via the same equations and axioms, merely submitted to different realizations, with a similar result holding between Galilei's relativity and its isotopic covering,* as illustrated, e.g., by the abstract identity of the light isocone with the conventional cone, Eq. (2.13), the preservation on isospacetime of the speed of light in vacuum as the maximal causal speed, and other features.

Regrettably, we cannot possibly review these studies. Nevertheless, it is evident that an abstract mathematical presentation would lead to possible misinterpretations in the applications of isorelativity to scattering processes or prevent them altogether. Therefore, for minimal self-sufficiency

of these papers as well as to specialize the new laws to the interior of the scattering region, we recall the following isoaxioms in their *projection* on conventional spacetime restricted to the third space direction and time (see Refs. [13] for the general case):

ISOAXIOM I: The maximal causal speed within the scattering region is given by

$$\hat{V}_{max} = c \times \frac{n_3}{n_4} = C \times n_3. \tag{2.20}$$

ISOAXIOM II: The addition of speeds within the scattering region follows the isotopic law

$$V_{tot} = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2} \times \frac{n_4^2}{n_3^2}}. \tag{2.21}$$

ISOAXIOM III: The dilation of time, the contraction of space and the variation of mass with speed within the scattering region follow the isotopic laws

$$t' = \hat{\gamma} \times t, \tag{2.22a}$$

$$\ell' = \hat{\gamma}^{-1} \times \ell, \tag{2.22b}$$

$$m' = \hat{\gamma} \times m, \tag{2.22c}$$

ISOAXIOM IV: The frequency shift within the scattering region follows the isotopic law

$$\omega' = \hat{\gamma} \times [1 - \hat{\beta}\, cos(\hat{\alpha}) \times \omega, \tag{2.23}$$

ISOAXIOM V: The mass-energy isoequivalence within the scattering region follows the isotopic law

$$\hat{E} = m\, V_{max}^2 = m \times c^2 \times \frac{n_3^2}{n_4^2} = m \times C^2 \times c_3^2. \tag{2.24}$$

A few comments are now in order. It should be stressed that, at the pure mathematical level, e.g., when formulated on isospace over isofields, the above isoaxioms coincide with conventional axioms. Consequently, *the isoscattering theory preserves Einstein's axioms everywhere in the exterior and the interior region, and merely uses a broader realization of the*

same axioms for the interior case. As we shall see in the next section, we have the same occurrence in regard to the applicable mechanics because relativistic hadronic mechanics also coincides with relativistic quantum mechanics at the abstract mathematical level by conception and construction.

Interested readers are then suggested to inspect: the unique and unambiguous derivation of Isoaxioms I-V from the LPS isosymmetry [12,13,16], and the proof of their "direct universality" by J. V. Kadeisvili [41], A. K. Aringazin [42], and others. Therefore, *Isoaxioms I-V are the only known directly universal axioms for interior conditions that are invariant under a universal isosymmetry for arbitrary speeds of light.*

Rather unexpectedly, the above studies have established the *necessity of abandoning the speed of light as the maximal causal speed within physical media* because of unsurmontable difficulties in the use of the speed of light it in vacuum, the understanding being that the speed of light is indeed regained as maximal causal speed in empty space and isospace.

As an illustration, the use of special relativity within water leads to insufficiencies of one or the other axioms, e.g., if one assumes in water the maximal causal speed in vacuum, causality is preserved when electrons travel in water faster than the local speed of light, but the relativistic addition of speeds is violated. Vice-versa, the assumption of the speed of light in water as the maximal causal speed in water preserves the relativistic law of addition of speeds, but violates causality.

By comparison, isorelativity resolves these insufficiencies. In fact, water can be safely assumed as being homogeneous and isotropic. As a consequence, $n_3 = n_4$, and the maximal causal speed in water is the speed of light in vacuum, $V_{max} = c$, thus verifying causality, while the isorelativistic law of addition of speeds (4.21) is also verified [12,13,16].

A serious knowledge of the isoscattering theory requires a study of the experimental verification of isorelativity at the classical and operator levels presented in various papers and reviewed in monograph [16d], Chapter 5 of ref. [27] and paper 47. Some of the most important experimental verifications of relativistic quantum mechanics will be indicated in Section 3. An important, classical, verification has been that of Isoaxiom IV on the shift of light within physical media first presented in paper

[47], and indicated in more details in the next section. Additional direct verifications are under way based on the repetition within physical media of the historical experimental verifications of special relativity all that were notoriously conducted in vacuum.

The specialization of isorelativity to the iso-Galilean case is an instructive exercise for the interested reader [12].

2.5. Implications for the Scattering Region. It is now important to identify the implications of isorelativity, specifically, for the interior of scattering processes because said implications will eventually be verified or dismissed by experiments in due time. Among a variety of rather intriguing implications, we identify the following ones:

2.5.1) **Isotime.** The time in the interior of the scattering region is no longer the same as our time, and it is given by a quantity known as *isotime* with vast physical and epistemological implications. In fact, the light cone is recovered identically on isospacetime, Eq. (2.13), but under the lifting of time into the isotime,

$$t \to \hat{t} = t/n_4. \tag{2.25}$$

By recalling that, according to all known fits of isorelativity, the index of refraction within hadronic matter is smaller than one, $n_4 < 1$ [16d,27], the reconstruction of the speed of light in vacuum as the maximal causal speed in interior conditions implies that *the scattering region is represented on isospacetime over isofields via an isotime generally in the future with respect to our time,*

$$\hat{t} = t/n_4 > t,\ n_4 < 1,\ C > c. \tag{2.26}$$

Readers with a technical knowledge of the Lie-Santilli isotheory will see that the replacement in the interior scattering region of our time with isotime is *necessary* to reach the universal invariance of the locally varying speed of light, since said invariance is based on the lifting of spacetime units, including that of time. Alternatively, the use of the isotime for interior conditions is necessary to avoid the inconsistency theorems.

2.5.2) **Isoshift**, For the case of null angle of aberration, and for $v = |v| << c$, the conventional Doppler's shift can be well approximated with the expression

$$\omega' = (1 \pm \frac{v}{c} + ...) \times \omega, \tag{2.27}$$

where the sign $-$ represents motion of the source *away* from the observer, in which case we have the *Doppler redshift,* and the sign $+$ represents motion of the source *toward* the observer, in which case we have the *Doppler blueshift.* It is evident that *special relativity predicts no shift of the frequency of light of any type for* $v = 0$,

The predictions of isorelativity are considerably different than the above. In fact, under the same assumptions as above, isolaw (2.13) can be written in good approximation

$$\omega' = (1 \pm \frac{v}{c} \times \frac{n_4}{n_3} + ...) \times \omega. \tag{2.28}$$

The most important implication, identified since 1991 [12], is that *isorelativity predicts a shift of the frequency of light even in the absence of any relative motion between the source, the medium and the observer,* called *isoshift.* This is due to the fact that the characteristic functions are generally dependent on the velocities (Section 2.2). Consequently, it is possible to have a dependence of the type $n_3/n_4 = v \times f(x, \xi, \omega, ...)$ under which expression (4.28) becomes

$$Lim_{v=0}\, \omega' = (1 \pm f(x, \xi, \omega, ...) + ...) \times \omega. \tag{2.29}$$

in which case *the frequency shift is no longer necessarily null for null relative speeds,* thus implying the following cases:

2.5.2A) **Santilli isoredshift.** It occurs for physical media of low density and has been experimentally verified in air, for certain astrophysical bodies, and other interior conditions [47]. This case essentially implies that *light loses energy* $E = h\nu$ *when propagating within physical media of low density, thus experiencing a decrease of its frequency without relative motion.* This possibility has been used in Ref. [47] to indicate the possible absence of universe expansion, big bang and dark matter since they all

refer to motion of light within physical media in which the Doppler law not necessarily applies.

2.5.2B: **Santilli isoblueshift.** It is predicted for physical media of very high density, such as those in the interior of astrophysical bodies as well as in the interior of high energy scattering processes. In this case, we have the prediction that *light acquires energy $E = h\nu$ from the hyperdense medium, thus increasing its frequency.* Since all high energy scattering regions are hyperdense, *isorelativity predicts that photons detected outside the scattering region are isoblueshifted,* namely, they originate in the interior at a frequency smaller than that detected in the outside [47].

2.5.2C) **Doppler-Santilli isoshift.** It occurs when light propagates within a transparent physical medium but there exist a relative motion between the source, the medium and the observer, which relative motion remains fully relativistic. Note that there are several cases in which *the total shift can be null even though there exist a relative motion between the source and the observer,* since both, Doppler's and Santilli's shifts can be positive or negative depending on the case at hand.

2.5.3) **Isoequivalence.** Maximal causal speeds bigger than c are prohibited by special relativity because at the value $v = c$ the conventional axioms diverge, as it is the case for the mass

$$: im_{v=c} m' = Lim_{v=c} \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}} = \infty. \tag{2.30}$$

This is no longer the case for isorelativity because it does not predict infinities at $v = c$ due to the indicated dependence of the characteristic quantities on the speed, for which we may have the expression for the mass

$$Lim_{v>c}\, m' = \frac{m}{\sqrt{1 - f(x, \xi, ...)}}. \tag{2.31}$$

Consequently, *Santilli isorelativity allows within physical media maximal causal speeds bigger than the speed of light in vacuum,* with far reaching implications.

For instance, Einstein's historical energy equivalence $E = m \times c^2$ was limpidly referred to *point-like particles in vacuum,* under which conditions it has received vast experimental verifications. However, the same equivalence principle has never received experimental verifications for extended and hyperdense particles, for which it is merely assumed as being valid, again, since special relativity does not allow any alternative formulation.

By contrast, isorelativity allows unlimited maximal causal speeds and the LPS isosymmetry uniquely and unambiguously predicts isoequivalence (2.24). By recalling again that all fits of experimental data available to date suggest causal speeds inside hadronic media bigger than c, *the energy equivalence of the scattering region predicted by isorelativity is bigger than that predicted by the conventional scattering theory,*

$$\hat{E} = m \times c^2 \times \frac{n_3^2}{n_4^2} > m \times c^2, \tag{2.32}$$

It should be indicated that isoequivalence (2.32) has been used in Ref. [47] to indicate the possible absence of dark energy due to the prediction of a total energy in the universe much bigger than that predicted by special relativity, since the maximal causal speed in the interior of black holes and other astrophysical bodies is predicted as being much bigger than c. The possible verification in scattering experiments of the isoblueshift would evidently confirm isoequivalence (2.24) and, therefore, the absence of dark energy.

2.5.4) **Isoparticles.** One of the most insidious misrepresentations for the isoscattering theory is the use of the conventional notion of particle. As it is well known, particles are generally referred to as irreducible unitary representations of the LP symmetry. However, such a symmetry is assumed as being inapplicable for the interior of the scattering region by central assumption, thus leading to major inconsistencies when using the conventional notion of particle under isotopies. Rather insidiously, these inconsistencies often remain undetected by non-experts in the new field.

Another implication of isospacetime mutation is that, *in the transition from motion in vacuum to motion within hyperdense scattering regions, particles experience a mutation of their "intrinsic characteristics,"*

also called isorenormalization resulting in the notion of isoparticles characterized by irreducible isounitary representations of the covering LPS isosymmetry [12,13,16]. The covering notion of isoparticle requires an in depth knowledge of *regular and irregular isorepresentations of Lie-Santilli isoalgebras,* that have been illustrated in Section 1.9 for the case of spin $1/2$.

In essence, conventional renormalizations, those characterized by action-at-a-distance, potential interactions, leave unchanged the intrinsic characteristics of particles. However, when passing to the broader non-Hamiltonian interactions, their nonunitary structure causes an isorenormalization of all characteristics of particles, including intrinsic features, via the mechanism of alteration of Hamiltonian eigenvalues pointed out in Paper I, and more technically identified in Section 1.9 as occurring under irregular isosymmetries. Note that isorenormalizations are fully in line with isoshifts.

Experimentally, the evidence supporting the mutation of intrinsic characteristic of particles in interior conditions is rather vast, and includes particle physics. nuclear physics, chemistry and astrophysics [*loc. cit.*]. Theoretically, the quantitative study of mutations of intrinsic characteristics of particles is perhaps the most fascinating of the isoscattering theory because, contrary to popular beliefs in the 20th century, our knowledge of the irreducible representations of the abstract axioms of the Lorentz symmetry is at its infancy, as illustrated by their current vastly unknown irregular isorepresentations.

As we shall see in subsequent papers, the quantitative treatment of the isorenormalization of particles inside the scattering region is one of the most challenging problem for a serious appraisal of the isoscattering theory on mathematical, theoretical and experimental grounds.

2.5.5) **Isogravitation.** As indicated earlier, the Riemannian formulation of gravitation has no appreciable impact in the conventional scattering theory, and any attempt at its inclusion is faced with very serious consistency problems, such has the unavoidable lifting of the scattering theory to nonunitary forms, loss of quantum mechanics, lack of conservation over time of the numerical predictions, and other problems [48].

By contrast, a direct, axiomatically consistent impact of gravitation in

scattering processes becomes unavoidable for the covering isoscattering theory. This important aspect can be initially seen from isoinvariant (2,4) showing that *the Minkowski-Santilli isogeometry and related isospacetimes includes as particular cases the (pseudo-) Riemannian, Finslerian and all other possible geometries and related spacetimes in* $(3+1)$*-dimensions* [37].

The transition from the conventional to the isotopic formulation of gravity is provided by the following steps first proposed in Ref. [44] of 1994

I) Factorizing any possible (nonsingular, pseudo-) Riemannian, Finslerian, or other metric $g(x,...)$ into the *Minkowskian* metric η and a 4×4 matrix $\hat{T}_{gr}(x,...)$,

$$g(x,...) = \hat{T}_{gr}(x,...) \times \eta, \tag{2.33}$$

II) Introducing the *gravitational isounit* as the *inverse* of the matrix $\hat{T}_{gr}(x,...)$,

$$\hat{I}_{gr}(x,...) = \hat{T}_{gr}(x,...)^{-1}. \tag{2.34}$$

III) Reconstructing the Minkowskian geometry, the LP symmetry and special relativity with respect to the above gravitational isounit.

Since $\hat{T}_{gr}(x,...)$ is necessarily positive-definite for all nonsingular Riemannian, Finslerian or other metrics in $(3+1)$-dimensions, the resulting LPS isosymmetry is isomorphic to the conventional LP symmetry, thus allowing the treatment of gravitation with all the formulations studied so far in these papers, as well as those we shall study in the future. The resulting formulation of gravity is today known as *Santilli isogravitation.*

As an illustration, the celebrated Schwarzschild line element in the coordinates (θ,ϕ,r,t) admits the following *identical* reformulation as the isometric in isospacetime

$$ds^2 = r^2(d\theta^2 + sin^2 d\theta^2 + d\phi^2) + (1 - \frac{2\times M}{r})^{-1} \times dr^2 -$$
$$-(1 - \frac{2\times M}{r} \times dt^2 \equiv \hat{T}_{sch} \times \eta \equiv \hat{\eta}, \tag{2.35}$$

with *gravitational isounit and isotopic element*

$$\hat{T}_{sch} = Diag.[1,1,(1 - \frac{2\times M}{r})^{-1}, (1 - \frac{2\times M}{r})], \tag{2.36a}$$

$$\hat{I}_{sch} = Diag.[1, 1, (1 - \frac{2 \times M}{r}), (1 - \frac{2 \times M}{r})^{-1}], \qquad (2.36b)$$

where one should note the positive-definiteness of the gravitational isounit and we assume the reader is aware from Paper I of the need for isotrigonometry in the isotopic reformulation, hereon tacitly assumed.

The implications at large of the above formulation of gravitation are far reaching, and their specializations to scattering processes should be at least summarily outlined here due to their significance, such as the clear prediction presented in Paper III that very high energy scattering experiments can indeed generate mini-black-holes.

Let us begin our short outline with the following important

LEMMA 2.5.1 [40,44]: The isotopic reformulation of the Riemannian gravitation implies the loss of curvature in favor of the isoflatness of the Minkowski-Santilli isogeometry.

This fundamental result can be seen in a variety of ways, e.g., from the fact that, by conception and construction outlined in Section 2.2, the Minkowski-Santilli isogeometry is locally isomorphic to the *Minkowski* geometry, thus prohibiting any conventional notion of curvature. Alternatively, one can see the loss of curvature on a conceptual basis by noting that gravitation is entirely contained in the isotopic element $\hat{T}_{gr}$. Consequently, the deformation of the Minkowski metric caused by gravitation

$$\eta \ \rightarrow \ \hat{T}_{gr} \times \eta = \hat{\eta}, \qquad (2.37)$$

is compensated by the *inverse* deformation of the unit

$$I = Diag.(1, 1, 1, 1) \ \rightarrow \ \hat{I}_{gr} = (\hat{T}_{gr})^{-1}, \qquad (2.38)$$

without altering the original flatness in view of the novel isodilation symmetry of the Hilbert space, Eq. (3.19) of Paper I, the new isodilation invariance (2.18), or the very structure (2.9) spacetime invariants. In turn the loss of curvature in favor of isoflatness has the following implications rather important for scattering processes:

2.5.5A) **Consistent operator form of gravitation.** As it is well known, a consistent operator formulation of the Riemannian gravitation acceptable by the scientific community at large has not been achieved in one century of efforts due to unsurmontable problematic aspects or sheer inconsistencies caused by curvature, the ensuing nonunitary character of the theory, lack of the PCT theorem and other problems [48]. By comparison, isogravitation admits an axiomatically consistent operator formulation first achieved in ref. [44] merely given by embedding gravity in the *unit* of relativistic quantum mechanics, thus preserving its abstract axioms, and ensuing consistency, including the correct formulation of the PCT theorem and all that. Note that this result *cannot* be achieved with the Riemannian curvature (see Ref. [16c] for details).

2.5.5B) **Universal invariance of gravitation.** As it is well known, the conventional Riemannian formulation of gravitation solely admits a '"covariance." But its structure is notoriously noncanonical, thus activating the theorems of catastrophic inconsistency (see Ref. [48] for details). By comparison, the isotopic formulation of gravity admits the universal LPS isoinvariance with the resolution of said inconsistency problems. Again, the reader should keep in mind that the invariance of gravitation is impossible with the Riemannian curvature (see Refs. [37.44.16c] for details).

2.5.5C) **Unification of the Minkowskian and Riemannian geometries.** Traditionally, the Minkowskian and Riemannian geometries are differentiated, as it should be, when formulated on conventional spaces over conventional fields. However, the use of isospaces over isofields has allowed the unification of these two geometries into one single geometry, the Minkowski-Santilli isogeometry, and their differentiation via different isounits first achieved in Ref. [40]. But the isometric $\hat{\eta}(x, ...)$ has an explicit dependence on coordinates and other variables. Consequently, the Minkowski-Santilli isogeometry admits the entire machinery of the Riemannian geometry., such as covariant derivative., Christoffel's symbols, etc. only isotopically reformulated, with consequential *geometric unification of special and general relativities.* This result has the consequence, rather important for scattering processes, according to which *the*

Einstein-Hilbert field equations are preserved and identically reformulated in an invariant operator version for the interior of the scattering region. Note again that this result would be inconsistent under a Riemannian curvature on a number of grounds [48].

2.5.5D) **Isotopic grand unification.** It is equally well known that a grand unification of electroweak and gravitational interactions in a form acceptable by the scientific community at large has escaped all efforts beginning with Einstein. It is today known that the difficulties originate from: A) Inconsistencies in unifying a theory possessing an invariance with another theory solely possessing covariance (due to the activation by the latter of the inconsistency theorems for the entire unification [44]); B) Inconsistencies in unifying an operator theory on a flat spacetime with another on a curved spacetime (due to the ensuing nonunitary structure and activation, again, of the inconsistency theorems); and C) Inconsistencies in unifying a theory with full democracy between particles and antiparticles with a gravitational theory insufficient for the description of antiparticles. e/g/., without any distinction whatsoever between neutral particles and antiparticles (see Section 2.5). Thanks to the removal of curvature and the achievement of an invariant operator formulation, isogravity has resolved insufficiencies A, B, C, resulting in an axiomatically consistent *iso-grand-unification* in which gravitation is embedded in the unit of electroweak theories, first achieved in Refs. [45,46] (see monograph [15] for a comprehensive presentation including the necessary gravitational treatment of neutral or charged antimatter).

2.5.5E) **Interior gravitation.** As indicated in Paper I, prior to Einstein's time, there was a clear differentiation between exterior and interior problems. In fact, Schwarzschild wrote *two* papers, the first one for the *exterior gravitational problem* with his historical metric (2.35) and a vastly ignored second paper on the *interior gravitational problem.* The distinction between exterior and interior problems was then ignored for about one century via the abstraction of the latter problems to isolated point-particles in vacuum. The No Reduction Theorems reviewed in Paper I have suggested a return to the full differentiation between exterior and interior gravitational problems, thus relegating metric (2.35) to the

meaning intended by its originator, namely, for the exterior problem only. The advent of isogravitation has permitted significant advances in interior gravitational; problems, e.g., by achieving for the first time a direct geometric representation (that is, a representation via the isometric) of the locally varying speed of light, the density of the interior medium and other features. Gravitational collapse is then represented with the *limit of null value of the space component of the isounit and the limit to a divergent value of its time component,* as geometrically expected in any case, i.e.,

$$\hat{I}^{int}_{gr,space}(x, v, \xi, \omega, \psi, \partial\psi, ...) \to \infty, \tag{2.39a}$$

$$\hat{I}^{int}_{gr,time}(x, v, \xi, \omega, \psi, \partial\psi, ...) \to 0. \tag{2.39b}$$

To understand the above reformulation of gravitational collapse, one should keep in mind that *isotopic rules (2.39) are equations that can be solved not only in the coordinates, as it is the case for the Schwarzschild metric (2.35), but also in the velocities and other variables as it is necessary for realistic models of interior gravitation.* The issue as to whether a true singularity such as the notion of *black hole,* is preserved by interior isogravitation, or we merely have a gravitational collapse without singularity, such as the notion of *brown hole,* is under study at this writing and the outcome will be reported in a future paper.

3. Deformations-Isotopies of Mechanics

3.1. Introduction. The central dynamical problem of the isoscattering theory is that at the foundation of hadronic mechanics, namely, the achievement of a consistent operator formulation of nonconservative systems, called in these papers non-Hamiltonian systems. We are here referring to a problem that remained unsolved for most of the 20th century physics, whose solution required several decades of trials and errors. We believe it is important to outline the main points of this scientific journey so as to avoid potential misrepresentation of the proposed isoscattering theory in the event based on the older literature, or the use of mechanics that have been proved as being inapplicable.

A considerable number of applied mathematicians, theoretical physicists and experimentalists have contributed to the deformations-isotopies

of classical and quantum mechanics to represent non-Hamiltonian systems, including in chronological order from 1978 to 1983: R. M. Santilli, H. C. Myung, S. Okubo, J. Fronteau, A Tellez-Arenas, R. Mignani, A. O. E. Animalu, J. A. Kobussen, Y. Ylamed, N. Salingaros, T. Giill, A. J. Kalnay, H. Rauch, G. Eder, P. Caldirola, R. Trostel, A. Schober, R. J. Slobodrian, J. Sun, A. de Wet, A. D. Jannussis, G. Brodimas, D. S. Sourlas, N. Salingaros, N. Tsagas, D. P. K. Ghikas, E. Kapushik, F. Rohrlich, J. Snyaticki, N. Salingaros, P. Truini, G. Cassinelli, G. Lochak, D. Y. Kim, J. Salmon, M. Grmela, E. Tontio, J. G. Gilson, V. K. Agrawala, W. H. Steeb, M. Mijatovich, R. Broucke, and numerous others in the subsequent years. Regrettably, we cannot possibly review the studies by so many authors and have to restrict our outline to the aspects directly relevant for the isoscattering theory (see the comprehensive bibliography in Volume [16a]).

The insufficiency that prevented the achievement of a consistent operator formulation of non-Hamiltonian systems for such a long time was the absence of their universal representation via an action principle due to the crucial role of the latter for a consistent map to operator forms.

The scientific journey initiated with Volume [10a] in which it was established that non-Hamiltonian systems in two or more dimensions generally violate the integrability conditions for the existence of a Hamiltonian, the *conditions of variational selfadjointness.* Consequently, non-Hamiltonian systems in more than one dimension do not admit an analytic representation via the conventional *Hamiltonian action principle.*

The scientific journey then continued with Volume [10b] achieving the '"direct universality" for the representation of all well behaved non-Hamiltonian systems via the *Birkhoffian action principle.* However, the latter action principle resulted in being inapplicable for the needed operator map for various reasons, such as the fact that the emerging "wavefunctions" would depend on both coordinates and momenta, $\psi = \psi(t, r, p)$, thus having a structure beyond our current knowledge of operator mechanics. and departing in any case from the abstract axioms of quantum mechanics.

Following the abandonment of the Birkhoffian mechanics, numerous additional mechanics were attempted (as listed, e.g., in the 1991 edition of Refs. [13]), but they all had a noncanonical time evolution formulated

on conventional spaces over conventional fields, thus activating the inconsistency theorems of Paper I.

The needed breakthrough finally occurred with the discovery in memoir [7] of 1996 of the *isodifferential calculus* that permitted the achievement of the needed axiom preserving isotopies of the classical Hamiltonian mechanics with a consistent map to the operator image that resulted indeed to be an isotopy of quantum mechanics, as desired.

There was a similar impass in the construction of hadronic mechanics. Following the original proposal [5] with the basic Lie-isotopic and Lie-admissible dynamical equations, all possible isotopies and genotopies of the Hilbert space, functional analysis, Lie algebras, etc., were reached by the early 1980s. Nevertheless, hadronic mechanics failed to be invariant, thus verifying the inconsistency theorems of nonunitary theories. Additionally, and perhaps most insidiously, hadronic mechanics missed consistent isotopies and genotopies of the Schrödinger realization of the linear momentum, thus prohibiting practical applications, e.g., because of the lack of a consistent formulation of the angular momentum.

The needed breakthrough occurred, again, with memoir [7] thanks to the discovery of the isotopies and genotopies of the differential calculus that finally permitted, after two decades of failed attempts, the achievement of consistent isotopies and genotopies of the linear and angular momenta, with consequential basic invariance over time.

In conclusion, readers interested in a serious study of the isoscattering theory should be aware that the various classical and operator mechanics existing in the literature prior to the generalized differential calculi of Ref. [7] are either inapplicable to scattering processes, or they verify the theorems of catastrophic inconsistencies.

3.2. Deformations-Isotopies of Newtonian Mechanics. The isoscattering theory is based on the *isotopic branch of hadronic mechanics,* also known as *isomechanics* [4,5, 13,16]. Its primitive notion is that of *closed-isolated non-Hamiltonian systems* introduced in monograph [10b] defined as systems with Hamiltonian (*variationally selfadjoint,* SA) and non-Hamiltonian (*variationally nonselfadjoint,* NSA) internal forces, yet verifying the con-

ventional ten total conservation laws of the Galilei and Lorentz-Poincaré symmetries. We are referring to the following class of Newtonian systems

$$m_k \times \frac{dv_k}{dt} - F_k^{SA}(t,r) - F_k^{NSA}(t,r,v,...) = 0, \;\; k = 2,3,4,...,N, \qquad (3.1)$$

whose NSA forces verify the closure conditions

$$\sum_k \mathbf{F}_k^{NSA} = 0, \qquad (3.2a)$$

$$\sum_k \mathbf{r}_k \bigodot \mathbf{F}_k^{NSA} = 0, \qquad (3,2b)$$

$$\sum_k \mathbf{r}_k \bigwedge \mathbf{F}_k^{NSA} = 0, \qquad (3.2c)$$

with trivial relativistic extension, under which the conventional ten conservation laws are automatically verified.

Prior to Refs. [10], it was popularly believed that only Hamiltonian systems verify the conventional ten conservation laws. Refs. [10] dispelled this unsubstantiated belief by showing that interior dynamical problems of isolated systems, such as Jupiter, do indeed include contact, nonconservative, non-Hamiltonian internal forces, as established by nature in any case.

Hence. it is important for this paper to clarify that the basic assumption of the isoscattering theory, the admission of contact non-Hamiltonian forces for the interior scattering region, originates at the primitive Newtonian level, as necessary from the *No Reduction Theorems* reviewed in Paper I.

Note that the verification of conventional total conservation laws is crucial for isomechanics and the isoscattering theory. In fact, when the system is open we have the broader *Lie-admissible genomathematics and genomechanics* [13,16].

The next step is the representation of systems (3.1) with the isotopic lifting of Newton's equations, first introduced in Refs. [7,13b] and known as *Newton-Santilli isoequations,* permitting the first known broadening

of Newton's equation since Newton's time for the representation of extended, nonspherical and deformable particles under SA and NSA forces.

Far from being trivial, the referred structural broadening of Newton's equation required a structural generalization of the entire mathematics of Newton's equations, including numbers, vector spaces, functional analysis, as well as the very differential calculus discovered by Newton himself (with Leibnitz).

Despite its diversification, this vast effort remained insufficient for the desired structural broadening of Newton's equation, that for the representation of *extended* particles, since such a representation is manifestly absent in Newton' s formulation. In fact, the background Euclidean geometry solely permits the representation of particles as being point-like. Maturity was finally achieved thanks to the construction by the mathematicians Gr. Tsagas, D. Sourlas, R. Falcon Ganfornina and J. Nunez Valdes of the all fundamental *isotopology* [29] that finally allowed the consistent representation of extended particles beginning at the primitive topological level.

The *Newton-Santilli isoequations*, first reached in Ref. [7], can be expressed for simplicity in the following form with the sole isodifferential in the velocities (see Ref. [13b] for the general form)

$$m \times \frac{\hat{d}\hat{v}}{\hat{d}t} - F^{SA} = m \times \frac{dv}{dt} - F^{SA} - F^{NSA} = 0. \tag{3.3}$$

with elementary solution for v constant and F^{NSA} not dependent explicitly on time (see [*loc. cit.*] for the general case)

$$\hat{I}_t = 1, \;\; \hat{t} = t, \;\; \hat{I}_r = 1, \;\; \hat{r} = r, \tag{3.4a}$$

$$\hat{I}_v = exp(\frac{t}{mv} F^{NSA}), \;\; \hat{v} = v \times \hat{I}_v. \tag{3.4b}$$

It is important for this paper to know that the main mechanism of the isoscattering theory, the embedding of the non-Hamiltonian interactions in a generalization of the basic unit, originates at the primitive Newtonian level. Note the emergence of a realization of the isounit via the exponent, an occurrence that will result in being rather general for the isoscattering theory, as shown in Paper III.

Note finally the mechanism of the representation of non-Hamiltonian Newtonian systems consisting in turning their NSA form when formulated on conventional Euclidean space over conventional fields, into an *identical,* fully SA form when formulated on Euclid-Santilli isospaces over isofields. Such an identical SA form is then at the foundation of the subsequent representation via an isoaction principle and ensuing consistent map to operator isomechanics. Intriguingly, the achievement of this new representation required a generalization of the very differential calculus Newton had to develop for his original formulation.

3.3. Deformations-Isotopies of Hamiltonian Mechanics. the next central methodological branch of isomechanics is the analytic representation of Eqs. (3.4) via a *variational isoaction principle* also introduced in Refs. [7,13], here expressed in its general form for space and time isotopies

$$\hat{\delta}\hat{A}^o = \hat{\delta}\int_{t_1}^{t_2}(\hat{p}_k\hat{\times}\hat{d}\hat{r}^k - \hat{H}\hat{\times}\hat{d}\hat{t}) = \hat{\delta}\int_{t_1}^{t_2}[p_k\times\hat{T}^k_{\hat{r},i}(t,r,p,...)\times\hat{d}\hat{r}^i - \hat{H}\times\hat{T}_{\hat{t}}\times d\hat{t}] = 0, \tag{3.5}$$

characterizing thhe *Hamilton-Santilli isoequations*

$$\frac{\hat{d}\hat{r}^k}{\hat{d}\hat{t}} = \frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{p}_k}, \quad \frac{\hat{\partial}\hat{p}_k}{\hat{d}\hat{t}} = -\frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{r}^k}. \tag{3.6}$$

that provide a direct analytic representation of NSA Newtonian systems (3.4), as well as the *Hamilton-Jacobi-Santilli isoequations* [*loc. cit.*]

$$\frac{\hat{\partial}\hat{A}^o}{\hat{\partial}\hat{t}^d} + \hat{H} = 0, \tag{3.7a}$$

$$\frac{\hat{\partial}\hat{A}^o}{\hat{\partial}\hat{r}^k} - \hat{p}_k = 0, \tag{3.7b}$$

$$\frac{\hat{\partial}\hat{A}^o}{\hat{\partial}\hat{p}_k} \equiv 0. \tag{3.7c}$$

. The latter expression, evidently at the foundation of operator maps, illustrate again the fundamental role of the isodifferential calculus for all deformations-isotopies.

Note the formal identity at the abstract level of the conventional Hamiltonian mechanics and its isotopic image. This illustrates that the abstract axioms of Hamiltonian mechanics have representational capability dramatically broader than those believed for centuries, although they can be seen only under the using the appropriate broader mathematics.

3.4. Deformations-Isotopies of Quantum Mechanics. The conventional naive quantization

$$A^o = \int_{t_1}^{t_2} (p_k \times dx^k - H \times dt) \to -i \times \hbar \times log\psi, \tag{3.8}$$

is lifted into the following *Animalu-Santilli isoquantization* [49] via the use of the isologarithm, Eq. (3.5) of Paper I,

$$\hat{A}^o = \int_{t_1}^{t_2} (\hat{p}_k \hat{\times} \hat{d}\hat{x}^k - \hat{H} \hat{\times} \hat{d}\hat{t}) \to -i \times \hat{log}\hat{\psi} = -i \times \hat{I} \times \hat{Log}\hat{\psi}, \tag{3.9}$$

with corresponding isotopies for the symplectic and other operator maps [13].

We are now equipped to present the operator image of the classical isotopies. The first nonunitary image of the Schrödinger equation was presented in the original proposal [5] to build hadronic mechanics, and then studied by several authors, such as Myung and Santilli [50], Mignani [51] and others (see the general bibliography in Ref. [16a]), although all these initial versions were formulated on conventional Hilbert spaces and/or over conventional fields, thus activating the inconsistency theorems reviewed earlier.

The axiomatically correct, and invariant, nonunitary isotopic image of Schrödinger equation was reached in Ref. [7] by applying map (3.9) to Eqs./ (3.7). The resulting equations are today known as *Schrödinger-Santilli isoequations* on $\hat{\mathcal{H}}$ over $\hat{\mathcal{C}}$ and can be written (see Refs. [13] for a

detailed treatment)

$$\hat{i}\hat{\times}\hat{\partial}_t\hat{\psi}(\hat{t},\hat{r}) = \hat{H}\hat{\times}\hat{\psi}(\hat{t},\hat{r}) = \hat{E}\hat{\times}\hat{\psi}(\hat{t},\hat{r}), \tag{3.10}$$

with *isolinear momentum,* first formulated in 1995 [*loc. cit.*]],

$$\hat{p}_k\hat{\times}\hat{\psi}(\hat{t},\hat{r}) = -\hat{i}\hat{\times}\hat{\partial}_k\hat{\psi}(\hat{t},\hat{r}), \tag{3.11}$$

canonical isocommutation rules

$$[\hat{r}^{\hat{i}},\hat{p}_j] = \hat{i}\hat{\times}\hat{\delta}^i_j, \;\; [\hat{r}^{\hat{i}},\hat{r}^j] = [\hat{p}_{\hat{i}},\hat{p}_j] = \hat{0}. \tag{3.12}$$

isonormalization

$$< \hat{\psi}|\hat{\times}|\hat{\psi} > \times\hat{I} = \hat{I}, \tag{3.13}$$

isoexpectation values of an iso-Hermitean operator $\hat{A}$

$$\hat{<}\hat{A}\hat{>} =< \hat{\psi}|\hat{\times}\hat{A}\hat{\times}|\hat{\psi} > \times\hat{I}, \tag{3.14}$$

and *isounit identities*

$$\hat{I}\hat{\times}\hat{\psi} = \hat{\psi}, \quad < \hat{\psi}|\hat{\times}\hat{I}\hat{\times}|\hat{\psi} >= \hat{I}. \tag{3.15}$$

assuring that $\hat{I}$ is indeed the correct basic unit of the theory (see Section 3 of Paper I for details).

The above equations are written on $\hat{\mathcal{H}}$ over $\hat{\mathcal{C}}$ and can be written in their *projection* on $\mathcal{H}$ over C

$$i \times \hat{I}_t(t,r,p,\xi,\omega,\psi,\partial\psi,...) \times \partial_t\hat{\psi}(t,r) =$$

$$= H \times T_r(t,r,p,\xi,\omega,\psi,\partial\psi,...) \times \hat{\psi}(t,r) = E \times \hat{\psi}(t,r), \tag{3.16a}$$

$$p_k \times T_r(t,r,p,\xi,\omega,\psi,\partial\psi,...) \times \hat{\psi}(t,r) =$$

$$= -i \times \hat{I}^i_k((t,r,p,\xi,\omega,\psi,\partial\psi,...) \times \partial_i\hat{\psi}(t,r), \tag{3.16b}$$

$$[r^{\hat{i}},p_j] = i \times \hat{I}_r \times \delta^i_j, \;\; [r^{\hat{i}},r^j] = [p_{\hat{i}},p_j] = 0. \tag{3.16c}$$

$$\hat{<}\hat{A}\hat{>} =< \hat{\psi}| \times T \times A \times T \times |\hat{\psi} > \times\hat{I}. \tag{3.16d}$$

We also have the following *isoplanewaves,* namely, conventional planewaves experiencing a mutation due to their immersion within the scattering region,

$$\hat{\psi}(r) = \hat{e}^{i\times k\times r} = \hat{I} \times (e^{i\times k\times T\times r}), \tag{3.17}$$

with isoeigenvalue equation (assuming for simplicity that the isounit does not depend on coordinates, see Ref. [13b] for the general case)

$$p \times T \times \hat{\psi} = -i \times \hat{I} \times \partial_r \hat{\psi} = -i^2 \times \hat{I} \times T \times k \times \hat{\psi} = k \times \hat{\psi}, \tag{3.18}$$

The reader can now see the fundamental relevance for hadronic mechanics of the isodifferential calculus because, until achieved, hadronic mechanics had no consistent formulation of the linear momentum, angular momentum, plane waves and other basic features.

The isotopies of Heisenberg equations were introduced in the original proposal [5] of 1978; studied by various authors, such as Myung and Santilli [51] and others (see Refs. [16a] for comprehensive bibliography); then reformulated in Ref. [7] via the isodifferential calculus; and are today known as *Heisenberg-Santilli isoequations.* They can be written in their general form on $\hat{\mathcal{H}}$ over $\hat{\mathcal{C}}$ for the finite time evolution of a (Hermitean) operator $\hat{A}$

$$\hat{A}(\hat{t}) = \hat{U}(\hat{t})\hat{\times}\hat{A}(\hat{0})\hat{\times}\hat{U}^{\dagger}(\hat{t}) = (\hat{e}^{i\times H\times t})\hat{\times}\hat{A}(\hat{0})\hat{\times}(\hat{e}^{-i\times t\times H}), \tag{3.19}$$

characterizing a one-dimensional *Lie-Santilli isotransformation group* (see Section 1.5) with corresponding infinitesimal form

$$\hat{i}\hat{\times}\frac{\hat{d}\hat{A}}{\hat{d}\hat{t}} = \hat{A}\hat{\times}\hat{H} - \hat{H}\hat{\times}\hat{A} = [\hat{A}\hat{,}\hat{H}]. \tag{3.20}$$

By using isoexponentiation (3.4), the above equations can be written in their projection on $\mathcal{H}$ over $\mathcal{C}$

$$A(t) = e^{i\times H\times T\times t)} \times A(0) \times (e^{-i\times t\times T\times H}), \tag{3.21a}$$

$$i \times \hat{I}_t((t, r, p, \xi, \omega, \psi, \partial\psi, ...) \times \frac{dA}{dt} =$$

$$= A \times T_r((t, r, p, \xi, \omega, \psi, \partial\psi, ...) \times H - H \times T - r((t, r, p, \xi, \omega, \psi, \partial\psi, ...) \times A =$$
$$= [A\hat{,}H], \tag{3.21b}$$

with fundamental isounitary property

$$\hat{U} = \hat{e}^{i \times H \times t}, \ \ \hat{U} \hat{\times} \hat{U}^\dagger = \hat{U}^\dagger \hat{\times} \hat{U} = \hat{I}_r. \tag{3.22}$$

This concludes our elementary review of the basic equations of the nonrelativistic isotopic branch of hadronic mechanics as minimally needed for the elaboration of the isoscattering theory. The elements of the relativistic extension are indicated in the next section.

We should recall from Section 3.5 of Paper I that isotopies preserve Hermiticity to the extent that the operations of Hermiticity and iso-Hermiticity coincide. Hence, all observables of quantum mechanics remain observables for the covering hadronic mechanics. Also, by conception and construction, quantum and hadronic mechanics coincide at the abstract, realization-free level. Therefore, any criticism on the axiomatic structure of hadronic mechanics is indeed a criticism on the axiomatic structure of quantum mechanics.

Additionally, we should also recall that *hadronic mechanics has been conceived and constructed as a kind of completion of quantum mechanics much along the historical Einstein-Podolsky-Rosen argument* (see Ref. [34] on the latter aspects. In this way, *the isoscattering theory can be considered a realization of one of the possible realizations of hidden variables with explicit and concrete identification of their realization, physical origin and meaning.*

By recalling the content of Paper I, isounitarity property (3.22), as well as the isolocality and isolinearity, establish the resolution of the inconsistency theorems for nonunitary theories. In particular, the isolinearity is crucial for the application of the mechanics to multiple scattering process, an occurrence with unresolved problematic aspects for a nonlinear formulation of the conventional scattering theory (see also Sections 3.4 and 3.5 of Paper I).

Note that the various "deformations of quantum mechanics" existing in the literature were formulated years following the appearance of the isotopies and genotopies [4,5] of 1978, and are essentially *identical* to

these original formulations formulated on conventional spaces over conventional fields, thus being catastrophically inconsistent for the reasons indicated earlier (see Refs. [6-12] of Paper I).

3.5. Deformations-Isotopies of Dirac's Equations. As it is well known, Dirac's equation plays a central role in Feynman's diagrams. Consequently, the covering isoscattering theory is crucially dependent on the deformations-isotopies of Dirac's equation.

Due to their importance, these isotopies were studied since the early stages of hadronic mechanics (see Ref. [12,13b] for original studies). In these papers, we shall use the first invariant isotopies of Dirac's equation achieved in Refs. [38,39] and used for the construction of the isotopies of the spinorial covering of the LP symmetry, as well as for the first known relativistic representation of all characteristics of the neutron in its synthesis from a proton and an electron in the core of a star (see Kaldeivili review [52]). The new equations are today known as the *Dirac-Santilli isoequations,* where we use the plural due to the variety of different realizations characterized by different isounits.

As it is also well known, the conventional Dirac equation represents *an electron in the external field of the proton.* The primary use of the covering isotopic formulation is that of representing a *mutated (isorenormalized) electron when immersed inside the proton also assumed as external.* The mutated electron was called *eleton* in the original proposal [5], but the name of *isoelectron* is nowadays more generally adopted due to the use of the prefix iso for protons and other particles when in interior conditions.

The isotopies of the Dirac equation belong to the branch of hadronic mechanics known as *relativistic isomechanics* (see memoir [53] for a review). Its foundations are given by the second-order iso-Casimir invariant (2.5b) that, via the use of the relativistic version of the isolinear momentum (3.11) characterizes the *isotopies of the Klein-Gordon equation* on $\hat{M}(\hat{x},\hat{\eta},\hat{\mathcal{R}})$ (see Ref. [13b] for detailed treatment of relativistic hadronic mechanics)

$$(\hat{\Xi}^{\mu\nu}\hat{\times}\hat{p}_\mu\hat{\times}\hat{p}_\nu - \hat{m}^{\hat{2}}\hat{\times}\hat{V}^{\hat{2}}_{max})\hat{\times}|\hat{\psi}(x)>=$$
$$= (\hat{\eta}^{\mu\nu}\times\hat{\partial}_\mu\hat{\partial}_\nu - m^2\times V^2_{max})\times|\hat{\psi}(x)>=0. \qquad (3.23)$$

as well as *relativistic isokinematics.*

The desired isotopies of Dirac's equation are then obtained via a linearization of the above equation similar to the conventional linearization, resulting in the following two cases:

CASE I: REGULAR DIRAC-SANTILLI ISOEQUATIONS.
In this case, the regular isotopies of spacetime can be described by the isounit, isotopic element and isometric (2.1) hereon denoted $\hat{I}_{st}$, T_{st}, and $\hat{\Xi}_{st} = \hat{\eta}_{st} \times \hat{I}_{st}$, respectively, while the regular isotopies of the spin are tose of Section 1.9 with isounit, isotopic element and isometric (1.33) hereon denoted $\hat{I}_{sp}$, T_{sp} and $\hat{\delta}_{sp}$.respectively.

By using again the notation of Eqs. (2.1b) and by assuming $\Gamma = \gamma \times \hat{I}_{st}$, the *regular Dirac-Santilli isoequations* can then be written [37.38]

$$(\hat{\Xi}_{st}^{\mu\nu} \hat{\times} \hat{\Gamma}_\mu \hat{\times} \hat{p}_\nu + \hat{i} \times \hat{m} \hat{\times} \hat{V}_{max}) \times \hat{T}_{st} \times |\hat{\psi}(x) >=$$

$$= (\hat{\eta}_{st}^{\mu\nu} \times \hat{\gamma}_\mu \times \hat{\partial}_\nu + i \times m \times V_{max}) \times |\hat{\psi}(x) >= 0. \tag{3.24}$$

where, by using isomatrices (1.36), the *regular isogamma matrices* have a structure of the type

$$\hat{\gamma}_k = b_k \times \begin{pmatrix} 0 & \hat{\sigma}_k \\ -\hat{\sigma}_k & 0 \end{pmatrix}, \;\; \hat{\gamma}_4 = i \times b_4 \times \begin{pmatrix} I_{2\times 2} & 0 \\ 0 & -I_{2\times 2} \end{pmatrix}, \tag{3.25}$$

with anti-isocommutation rules

$$\{\hat{\gamma}_\mu, \hat{\gamma}_\nu\} = \hat{\gamma}_\mu \times T_{st} \times \hat{\gamma}_\nu + \hat{\gamma}_\nu \times T_{st} \times \hat{\gamma}_\mu = 2 \times \hat{\eta}_{\mu\nu}, \tag{3.26}$$

The first implications for the isoscattering theory is that of *embedding gravitation directly in the basic metric $\hat{\eta}$ of the scattering region.* As an example, rules (3.26) can characterize (twice) the Schwarzschild metric as in Eqs. (2.35). However, the isometric can be more complex than that to include velocity-dependent internal effects, as well as the locally varying speed of light.

As one can see, in this particular case, the isotopies essentially offer *five* additional characteristic quantities, four for the spacetime mutation and one for the spin, that are available for the representation of experimentally measurable features of the scattering region, such as shape,

density and anisotropy (requiring precisely five values). Note that the representation of these features is essentially outside the capabilities of quantum scattering theories.

Reader can now see the comments on antimatter of Section 2.5 of Paper I, to the effect that *there exist no irreducible or reducible representation of the conventional $SU(2)$ spin algebra having the structure of Dirac's gamma matrices.* By contrast, the conventional gamma matrices are characterized by the Kronecker product of an irreducible representation for spin $1/2$ and its isodual, since the isodual unit $I^d_{2\times 2} = -I_{2\times 2}$ appears directly in γ_4 and Pauli's matrices verify the isodual rule $\sigma^d_k = -\sigma^\dagger_k = -\sigma_k$.

Consequently, *the conventional Dirac equation is reinterpreted as representing one electron and its antiparticle* without any need of the "hole theory" or second quantization, since the isodual theory of antimatter holds at the *Newtonian* level, let alone that of first quantization (see monograph [15] for comprehensive studies).

Evidently, the above features persist under isotopies, and we shall write in these papers the isogamma matrices in the form

$$\hat{\gamma}_k = b_k \times \begin{pmatrix} 0 & \hat{\sigma}_k \\ \hat{\sigma}^d_k & 0 \end{pmatrix}, \ \hat{\gamma}_4 = i \times b_4 \times \begin{pmatrix} I_{2\times 2} & 0 \\ 0 & I^d_{2\times 2} \end{pmatrix}, \tag{3.27a}$$

$$|\hat{\psi} >= \begin{pmatrix} |\hat{\psi}^- > \\ \hat{\psi}^+ > \end{pmatrix}, \tag{3.27b}$$

where the upper symbol d represents isodual conjugation (anti-Hermitean conjugation), and $|\hat{\psi}^- >, \hat{\psi}^+ >$ represent the two-components wavefunctions for the isoelectron and the isopositron under the respective external fields of a proton and an antiproton considered as external. In different words, Eq. (3.24) can represents the Kronecker product of a neutron and an antineutron [52].

CASE II: IRREGULAR DIRAC-SANTILLI ISOEQUATIONS.
In this case we have the same equations (3.24) and the same isogammas (3.25), but the Pauli-Santilli isomatrices are irregular, e.g., are given by matrices (1.41) representing a mutation (isorenormalization) of the spin.

An important implication of the irregular equations is that the total angular momentum of the isoelectron is no longer conventionally quantized, and can assume, in particular, the null value,

$$J_e = s + j = \frac{1}{2}, \frac{3}{2}, ... \to \hat{I}_{\hat{e}} = 0. \tag{3.28}$$

as needed for an invariant representation of the neutron spin in its synthesis from a proton and an electron [38,39,52].

Note that in the above case we solely have the mutation of the *angular momentum,* but not that of spin, the latter being expected under energies dramatically bigger than those for the synthesis of the neutron.

3.6. Dirac's Generalization of Dirac's Equation. The identification of the background methods for the treatment of the isoscattering theory would be grossly deficient without an outline of the generalization of Dirac's equation achieved by Dirac himself in two of his last, vastly ignored papers [54,55] admitting an essential isotopic structure as well as the null value of the total angular momentum so crucial for a quantitative representation of the neutron synthesis.

The *Dirac's generalization of Dirac equation* is given by [54]

$$(a_\mu \times \partial_\mu + \beta) \times q \times \psi = 0, \tag{3.29a}$$

$$q = \begin{pmatrix} q_1, p_1 \\ q_2, p_2 \end{pmatrix}, \ \ \psi = \begin{pmatrix} psi_{1+}, \psi_{1-} \\ \psi_{2+}, \psi_{2-} \end{pmatrix}, \tag{3.29b}$$

where the reader can immediately recognize the role of the q-quantity as characterizing a right isomodular action which is at the foundation of the Lie-Santilli isotheory, as well as of Eq. (3.24)

By assuming

$$a_4 = I_{4x4}, \tag{3.30}$$

Dirac's a-matrices are characterized by the expression

$$a_\mu \times \beta \times a_\nu + a_\nu \times \beta \times a_\mu = 2 \times \beta \times \eta_{\mu\nu}, \tag{3.31}$$

where the reader will immediately see the same isotopic structure of isoanticommutators (3.26), and $\eta_{\mu\nu}$ is the conventional Minkowski metric.

On the basis of the above structure, Dirac reached the following realization of the a- and β-matrices

$$\beta = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \ \ a_1 = i \times \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \tag{3.32a}$$

$$a_3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \ \ a_3 = i \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{3.32b}$$

The total angular momentum is characterized by

$$S_{ij} = -(a_i \times \beta \times a_j - a_j \times \beta \times a_i) \times \frac{q \times q^t}{8}, \tag{3.33}$$

where t stands for transposed, and possesses the eigenvalues

$$S^2 = S_{12}^2 + S_{23}^2 + S_{31}^2 =$$

$$= \frac{1}{8} \times (q_1^2 + p_1^2 + q_2^2 + p_2^2) = J \times (J+1), \tag{3.34a}$$

$$J = \frac{1}{4} \times (q_1 + p_1 + q_2 + p_2) - \frac{1}{2} = \frac{1}{2} \times (n + n'), \tag{3.34b}$$

$$n, \ n' \ = \ 0, 1, 2, 3, ... \tag{3.34c}$$

thus *admitting the value $J = 0$ for the ground state.*

The historical aspect particularly significant for hadronic mechanics at large, and the isoscattering theory in particular, is that, without his knowledge, *Dirac's generalization of Dirac's equation possesses an irreducible isotopic structure* with isounit and isotopic element first identified in Ref. [13b]

$$\hat{I} = \beta^{-1}, \ \ T = \beta, \tag{3.35}$$

where the irreducibility is referred to the property that papers [52,53] become inconsistent unless *entirely* elaborated with respect to the isoproduct

$$A \hat{\times} B = A \times T \times B, \ T = \beta. \tag{3.36}$$

We cannot close this section without the indication that, for structural consistency, Dirac's generalized equation cannot be formulated on the conventional Minkowski space $M(x,\eta,\mathcal{R})$ and must be formulated on the Minkowski-Santilli isospace $\hat{M}(\hat{x},\hat{\eta},\hat{\mathcal{R}})$, with isometric [13b]

$$\hat{\eta} = \beta \times \eta = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \tag{3,37}$$

namely, Dirac not only had the intuition without elaboration of the isotopic formalism, but identified without his knowledge the first known *nondiagonal realization of the spacetime isometric* (3.37).

Rather than being an innocuous occurrence, the implications are far reaching because the line element now reads

$$\hat{x^2} = x^\mu \times (\beta^\rho_\mu \times \eta_{\rho\nu}) \times x^\nu = x^\mu \times \hat{\eta}_{\mu\nu} =$$

$$= x^1 \times x^3 - x^2 \times x^4 - x^3 \times x^1 - x^2 \times x^4 = -2 \times x^2 \times x^4, \tag{3.38}$$

namely, when reformulated in an invariant way, *Dirac's isoequation (3.29) mutates spacetime from the conventional four-dimensions down to two-dimensions.*

The implications for the isoscattering theory are far reaching because, when the interior scattering region is characterized by Dirac's generalized equation, it loses completely the conventional four dimensions, by reducing spacetime solely to two dimensions, even though fully perceived from the outside as being four dimensional.

In summary, due to the power of his intuition, perhaps unprecedented in scientific history, Dirac should be consider the precursor of:

1) The isodual theory of antimatter, that originated from Dirac's negative unit $-I_{2\times 2}$ in the conventional γ_4 matrix [15];

2) The isotopic formalism and, consequently, the isoscattering theory, that originated from Dirac's generalized equation (3.29) [13b]; and

3) The first exact representation of the synthesis of the neutron from a proton and an electron in the core of a star requiring a null total angular

momentum of the electron, that was first achieved by Dirac in Eq. (3.34) [39].

As a personal note, at one of his last participations at scientific meetings in the early 1980s in Florida, Santilli briefly presented to Dirac the isotopic formalism with the connection to his papers [54,55] and the indication of the strong convergence of conventionally divergent series under the isotropic product $A \times T \times B - B \times T \times A, |T| \ll 1$, thus including the possible achievement of Dirac's dream for a scattering theory without divergences.

Contrary to his normally reserved nature, Dirac showed great interest. Unfortunately, Dirac's death in 1984 deprived the then newly born isotopies of his most powerful supporter, thus delaying collegial acceptance for decades. A rewarding aspect is that, with the power of his intuition, Dirac did in fact see prior to his death the technical feasibility of his dream of a scattering theory without divergencies hereon called the *Dirac's problem* (for more details in the episode, one may consult Ref. [16d]).

3.7. Experimental, Verifications. The presentation of the isoscattering theory in Paper III could be misleading to non-expert in the field without the indication of rather significant experimental verifications and novel applications of hadronic mechanics in various quantitative sciences along the following setting:

I) CONDITIONS OF EXACT VALIDITY OF QUANTUM MECHANICS. They are assumed as being those of special relativity, namely, the conditions of *exterior dynamical problems* (point-particles and electromagnetic waves propagating in vacuum), since the latter do admit the effective point-like abstraction of particles necessary for the validity of the local-differential and topological, foundations of the theory. Therefore, these papers assume that quantum mechanics is *exactly* valid for the structure of the hydrogen atom, particles in accelerators, crystals, and numerous other exterior problems.

II) CONDITIONS OF APPROXIMATE VALIDITY OF QUANTUM MECHANICS. These papers assume that quantum mechanics is only *approx-*

imately valid for *interior dynamical problems* (extended particles and electromagnetic waves moving within physica;l media) due to expected nonlinear, nonlocal and nonpotential interactions compared to the strictly linear, local and potential character of the mechanics. In particular, these papers assume that all arbitrary parameters and functions of unknown physical origin whose values are fitted from experimental data, represent in reality *deviations* from the very axioms of quantum mechanics, as illustrated by the following representative cases:

II-A) The inability by quantum mechanics to achieve an exact representation of the binding energy and other features of the hydrogen molecule, that required the use of the so-called "screened Coulomb potentials" of the type $V^*(r) = f(r) \times e^2/r$ with evident loss of quantized levels and other problematic aspects. By comparison, the use of hadronic mechanics has permitted the achievement of a numerically exact and invariant representation of the binding energy of the hydrogen and other molecules from first axiomatic principles without *ad hoc* functions of unknown physical origin or meaning. The representation is achieved via the sole admission of non-Hamiltonian interactions originating from the deep mutual penetration of the wavepackets of valence electrons, and provides convergent power series dramatically faster than those of quantum mechanics [14]. Similar insufficiencies of quantum mechanics exist for the representation of the simplest possible nucleus, the deuteron, not to mention very large deviations for heavy nuclei, for which the use of hadronic mechanics has provided distinct advances [16d,27].

II-B) The representation via relativistic quantum mechanics of the two-point functions of the Bose-Einstein correlation in a way compatible with experimental data requires *four* arbitrary parameters of unknown physical origin or meaning (called the "chaoticity parameters"), that are fitted from experimental data and relativistic quantum mechanics is then claimed as being valid. However, the vacuum expectation values of a diagonal two-dimensional Hamiltonian can at best allow *two* arbitrary parameters, thus indicating that the very structure of the Bose-Einstein correlation is beyond the representational capabilities of relativistic quantum mechanics. After all, said correlation originates from the deep mutual penetration and consequential annihilation protons and antiprotons

under which linear, local and potential treatments can at best be approximately valid. By comparison, relativistic hadronic mechanics achieves an exact and invariant representation of the Bose-Einstein correlation at both high and low energies via the four characteristic quantities of isospacetime, three of which represent the very elongated fireball and the forth represents its density [16d,27].

II-C) As it is equally well known, the representation of the behavior of the meanlives of unstable hadrons with speed is equally achieved via the introduction of a number of parameters, functions, expansions, cut off and other mechanisms to achieve compatibility with relativistic quantum mechanics. However, the latter theory is fully reversible over time, while spontaneous hadrons decays are strictly irreversible. Hence, the latter theory cannot possibly be exact for the former events. The use of the Lie-admissible branch of hadronic mechanics permits an exact representation of the $irreversible\ character$ of the decay in a way compatible with experimental data from first axiomatic principles without conceptual or axiomatic adaptations [16d,27].

III) CONDITIONS OF INAPPLICABILITY OF QUANTUM MECHANICS. Throughout the 20th century, it was widely believed that quantum mechanics can be applied to all possible conditions of particles existing in the universe. At a meeting in February 1978, Herman Feshback (then from MIT) confirmed to Santilli (then from Harvard) that Schrödinger's equation is inapplicable for the synthesis of the neutron from a proton and an electron as occurring in the core of stars, $p^+ + e^- \to n + \nu$, because the rest energy of the neutron is $0.782MeV\ bigger$ than the sum of the rest energies of the proton and the electron, thus requiring a $positive\ binding\ energy$ under which the indicial equation no longer admits physically acceptable solution. The origin of the missing energy from a possible relative kinetic energy between the proton and the electron has to be excluded due to their extremely small cross section at the indicated energy, and the same holds for the antineutrino in the conjugate reaction $p^+ + e^- + \bar{\nu} \to n$, thus establishing the $inapplicability$ (and certainly not the violation) of quantum mechanics for a quantitative treatment of the problem considered. At the same meeting, that signals the birth of

hadronic mechanics, Santilli indicated to Feshback that a nonunitary generalization of quantum mechanics in general, and of Schrödinger's equation in particular, does indeed admit physically meaningful for hadron syntheses with "missing energies" solutions for various reasons, e.g., the anomalous renormalization of the rest energy of the electron due to non-Hamiltonian interactions under which the binding energy returns to acquire the conventional $negative$ Coulomb form between opposite charges. The original proposal [4,5] published in April 1978 to construct hadronic mechanics provides a complete solution for the synthesis also requiring a "positive binding energy" $e^+ + e^- \to \pi^o$, while the full solution at the nonrelativistic and relativistic levels of the synthesis of the neutron was achieved after decades of study of the isotopies of Lie's theory and the discovery of the irregular isotopies of the spin symmetry outlined in Section 1 (see Ref. [52] for a general review).

We ended Paper I with the indication that, despite one millennium of studies, our knowledge of light is still at its infancy. It is appropriate to end this paper with the indication that quantum mechanics has been a beautiful episode in the history of physics but, in view of its evident limitations in face of the complexities of the physical world, any belief of its final character for all of nature causes the existing from the boundaries of science, the sole scientific issue, for which these papers have been written, being the selection via experiments, rather than personal or conceptual indications, of its appropriate structural generalization.

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NONUNITARY LIE-ISOTOPIC AND LIE-ADMISSIBLE SCATTERING THEORIES OF HADRONIC MECHANICS, III: Basic Lie-Isotopic Formulation without Divergencies

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Submission History

This paper and the remaining four of this series have been rejected by: G.t Hooft, editor of *Foundations of Physics;* L. Cifarelli, editor of *La Rivista Nuovo Cimento;* B. Nachtergaele, editor of *Journal of Mathematical Physics;* and E. Paal, editor of the *Journal of Generalized Lie Theories and Applications.* Additionally, Santilli stated *"I have not submitted the papers to APS and IoP journals due to their rejection of 'all' papers in the field since my last publications there in 1980* [PR D **22**, 892, 1980] *and 1983* [JoPG **18**, L141, 1983] *respectictively, all papers published in other refereed journals around the world.* In particular, t Hooft rejection was via a form letter without technical review; Cifarelli rejection included scientific wanderings plus the statement *"Moreover we do not wish receiving any further extension of your work.";* Nachtergaele blamed all JMP editors with the statement *"There was no support from the Editorial Board to publish the manuscripts."* (see the web site http://www.scientificethics.org/JMP.htm); and Paal's obstructed the very submission. The publisher submitted the papers to three outstanding physicists who released the following reviews: 1) *Following a lifelong research, Prof. Santilli has identified vast evidence on non-unitary effects in high energy inelastic scatterings requiring a re-inspection of 'experimental results' claimed via conventional unitary elaborations, with scientific, financial and ethical implications such that the suppression of the papers without due scientific process would constitute a crime against society;* 2) *Prof. Santilli's penetrating and impecccable analysis of scattering events establishes the need for the re-examination of basic paradigms due to experimentally established non-Lagrangian and non-Hamiltonian effects within the scattering region that must be debated in refereed publications for our community to verify minimal standards of ethic s and accountability;* 3) *Prof. Santilli's papers are: professionally written, technically outstanding, and potentially historical.* Therefore, without any consultation of the editors of the *Hadronic Journal,* the publisher elected to print the papers with the request to t Hooft, Cifarelli, Nachtergaele, Paal, APS-IoP editors and their friends that, in order to dispel increasing allegations of acting under orders by a scientific ring, they should publish their technical objections, if any possible, because science solely advances via publications and certainly not via equivocal 'reviews' voiced from opaque academic corridors. The series of five papers was finally accepted for publication by Kathmandu University, Nepal, following international review by qualified ethical scientists.The reasons for the political handling of these submissions are explained and denounced in the post by J. V. Kadeisvili Obscurantism at CERN on God's Particle? http://www.santilli-foundation.org/cern-obscurantism.php

This Submission History has been released by R. M. Santilli in his capacity as Founder and Honorary Editor in Chief of the Hadronic Journal without any consultation with any of the Editors.

Abstract

In the preceding Papers I and II, we have: introduced various arguments suggesting nonunitary coverings of the conventional unitary scattering; outlined the problematic aspects of nonunitary theories; presented the novel isomathematics allowing an invariant isounitary reformation; and specialized to scattering processes the isotopic branch of hadronic mechanics comprising the isotopies of Lies theory. special relativity and quantum mechanics. In this paper we present a solution of the Dirac legacy indicated in Paper II, namely, a nonunitary-isounitary scattering theory without divergencies ab initio indicated by one of the authors to Dirac prior to his death. Joint elaborations of measured quantities (cross section, scattering angles, etc.) via the conventional and isotopic scattering theories are presented in subsequent works to identify the expected implications in final experimental results of nonlinear, nonlocal and nonpotential effects in high energy scattering processes. As indicated in paper I, the presentation is restricted to reversible scattering processes, because the treatment of irreversible processes requires the yet broader Lie-admissible covering of the Lie-isotopic formulation.

Key words scattering theories, nonunitary theories, isounitary theories
PACS 03.65.Nk, 11.30.Cp, 03.65.-w

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Acknowledgments

References

1. Identification of the Isounit.

1.1. Introduction. In the preceding Refs. [1,2], herein indicated as Papers I and II, respectively, in which:

1) We presented various mathematical, theoretical and experimental arguments suggesting a re-inspection of the elaboration of measured quantities (cross sections, scattering angles, etc.) via the conventional, unitary, Hamiltonian scattering theory due to expected nonlinear, nonlocal and contact-nonpotential effects beyond the representational capabilities of quantum mechanics;

2) We pointed out that the sole possibility for a quantitative representation of the above effects, collectively called in these papers *non-Hamiltonian effects,* is the existing from the class of unitary equivalence of quantum mechanics, thus bringing again into focus past attempts at a nonunitary generalization of quantum scattering theories;

3) We then recalled the *Theorems of Catastrophic Mathematical and Physical Inconsistencies of Noncanonical and Nonunitary Theories* (Refs. [6-12] of Paper I) that have established the lack of invariance over time of units of measurements, the inability of predicting the same numerical values under the same conditions at different times, violation of causality and other inconsistencies under nonunitary time evolution;

4) We then introduced the elements, specialized to the scattering problem, of the novel *isomathematics* that has been specifically constructed for the invariant treatment of non-Hamiltonian effects and the resolution of the Inconsistency Theorems via the *isounitary reformulation of nonunitary scattering theories;* and

5) we finally presented the elements, also specialized to the scattering problem, of the *isotopic branch of hadronic mechanics* including the invariant deformations-isotopies of Lie's theory, special relativity and relativistic quantum mechanics, with emphasis on Dirac's pioneering work in the field.

We are now sufficiently equipment to initiate the presentation of the proposed covering *nonunitary-isounitary scattering theory* beginning with the identification of the all fundamental isounit.

To prevent major misrepresentations of the content of this paper, that often remain undetected by renowned experts in quantum mechanics but

non-experts in the covering hadronic mechanics, serious readers are suggested to acquire a technical knowledge of Papers I and II, hereon tacitly assumed, with particular reference to isofunctional analysis, isodifferential calculus, regular and irregular isorepresentations, regular and notions notions of spin, regular and irregular Dirac-Santilli isoequations, Dirac's generalization of Dirac's equation, and related aspects.

1.2. Main Conditions. As indicated in paper II, the isotopic branch of hadronic mechanics, also known as *isomechanics,* was built for the representation of the deep mutual penetration of the wavepackets and/or charge distributions of particles as occurring in the scattering region of Figure I.2. These conditions are of contact character, thus having zero range, and are expected as being nonlinear (in the wavefunctions), nonlocal (of integral type), and nonpotential (that is, not representable with a Hamiltonian). These non-Hamiltonian interactions are structurally beyond any possible representation by quantum mechanics due to its strictly linear, local-differential and potential-Hamiltonian character.

In view of these insufficiencies, the representation of the scattering region is done via *two* operators, the conventional Hamiltonian H as currently used in scattering theories, and Santilli's isounit $\hat{I}$ or isotopic element $\hat{T} = \hat{I}^{-1}$, for the representation of all non-Hamiltonian interactions and effects.

Following studies for decades, the isounit has been selected over any other alternative representations of non-Hamiltonian interactions because it is the only one assuring the crucial *invariance over time,* i.e., the characterization of the same numerical values under the same conditions at different times. After all, whether conventional or generalized, the unit is the fundamental invariant of all theories.

The main requirements for the isounit (or isotopic element) are the following (see Refs. [3] for detailed studies):

CONDITION I: Positive-definiteness.

$$\hat{I}(t, r, p, E, \xi, \omega, \psi, \partial\psi, ...) = \hat{I}^{\dagger}(t, r, p, E, \xi, \omega, \psi, \partial\psi, ...) =$$

$$1/\hat{T}(t, r, p, E, \xi, \omega, \psi, \partial\psi, ...) > 0. \tag{1.1}$$

This condition is sufficient to assure the preservation of the original axioms under isotopic liftings at all levels, with consequential local isomorphism between the Hilbert space and the Hilbert-Santilli isospace, the Lorentz-Poincaré (LP) symmetry and the Lorentz-Poincaré-Santilli (LPS) isosymmetry, quantum mechanics and isomechanics, etc.

CONDITION II: Elimination of quantum levels,

$$Lim_{r\gg 1fm}\hat{I}(t,r,p,E,\xi,\omega,\psi,\partial\psi,...) = \hbar,$$

$$Lim_{r\gg 1fm}\hat{T}(t,r,p,E,\xi,\omega,\psi,\partial\psi,...) = 1/\hbar. \qquad (1,2)$$

This condition assures the existence of a unique and unambiguous limit under which hadronic mechanics recovers quantum mechanics for all mutual distances of particles bigger than $1\ fm$, e,g,. for mutual distances bigger than the size of particles wavepackets. Also, Hamiltonian interactions remain fully valid inside the scattering region. Hence, the above condition clarifies the fact that isomechanics essentially provides expected *corrections* to quantum treatments in the scattering region (only).

A primary function of condition (1.2) is to illustrate the main feature of hadronic mechanics, *the absence of conventionally quantized energy levels within a hyperdense hadronic medium.* Conceptually, this condition is illustrated by the evident impossibility that an electron in the core of a star (or, equivalently, in the interior of a high energy scattering region) cannot possibly have the same quantized levels as occurring when orbiting in vacuum around the nucleus in the hydrogen atom. The isounit then represents the integro-differential *deviations* from conventional quantum levels caused by the medium.

Physically, condition (1.3) is illustrated by the so-called "screened Coulomb potentials" used in quantum chemistry, namely, the multiplication of the Coulomb potential $V = e^2/r$ by an arbitrary function, $V^* = f(r)e^2/r$ that has resulted in being necessary for a numerically exact representation of the mutual penetration of valence electron pairs in molecular structures. However, in so doing, it is evident that the screened potential no longer admits the quantum energy levels of the conventional potential, as studied in detail in monograph [4].

CONDITION III: Elimination of quantum divergencies,

$$||\hat{I}|| \gg 1, \quad ||\hat{T}|| \ll 1. \tag{1.3}$$

As it is well known to experts in the field, the above condition assures that all perturbative and other series that are divergent (or weakly convergent) for quantum mechanics are turned into strongly convergent series in the covering hadronic mechanics.

In turn, this important feature, whose achievement escaped the best minds of 20th century physics, implies *numerical differences* between the sum of divergent quantum series (turned into convergent forms via cut-off, arbitrary parameter of unknown physical origin, and other manipulations) and the naturally convergent hadronic series. Moreover, these differences are expected to produce numerical differences between the elaboration of experimental data via scattering and isoscattering theories, a feature well known to experts in hadronic mechanics since the early 1980 (see Section 1 of paper I and references quoted therein), but often ignored by particle physics to their peril. Note that conditions (1.3) are fully compatible with conditions (1.2), as shown by all realizations assumed later on.

1.3. Explicit Realization. All isounits used in experimental verifications of hadronic mechanics to date (see Refs. [3d]) have emerged as verifying quite naturally Conditions I, II and III that, therefore, *are not* subsidiary constraints, but seemingly natural occurrences.

Inspired by these experimental verifications, particularly for the representation of the Cooper pair and valence bonds [5], one can see the adoption of the following *realization of the isounit for the relativistic isoscattering theory*

$$\hat{I} = Diag.(n_1^2, n_2^2, n_3^2, n_4^2) \times e^{N \times \frac{\psi^i(x)}{\psi^f(x)} \times \int d^4x \times \psi^{i\dagger}(x) \times \psi^f(x)}, \tag{1.4}$$

where

$$n_\mu = n_\mu(t, r, p, E, \xi, \omega, \psi, \partial\psi, ...) > 0, \ \mu = 1, 2, 3, 4, \tag{1.5}$$

where:

1) The characteristic quantities $n_k,\ k = 1, 2, 3$ provide the first known representation of the *generally nonspherical and deformable shape* of the scattering region normalized to the values $n_k = 1, k = 1, m2, 3$ for the sphere;

2) The characteristic quantity n_4 provides the first known representation of the *density of the scattering region* (i.e.m the ratio between its energy and its volume) normalized to the value $n_4 = 1$ for the vacuum; and

3) the first known representation of the *inhomogeneity of the scattering region* is provided by a functional dependence of the characteristic quantities, e.g., on the local coordinates, while the first known representation of the *anisotropy of the scattering region* is provided, e.g., by different values of space and time characteristic quantities.

It should be indicated that the characteristic quantities have an explicit functional dependence when formulated in the *interior* of the scattering region, but they are generally averaged into constants when inspected from the outside, an assumption we shall tacitly make hereon.

It is easy to see that isounit (1.4) naturally verifies Conditions I, II, III. Additionally:

A) The integral $\int d^4x \times \psi^{i\dagger}(x) \times \psi^f(x)$ represent the intended nonlocal character of the scattering region as well as the verification of limits (1.2);

B) The ratio between initial and final wavefunctions, ψ^i/ψ^f, characterizes the intended nonlinear character of the scattering theory (with the possibility of additional embedding of nonlinear terms);

C) N is an isorenormalization constant to be identified later on;

and the *exponential* character of the isounit originates at the primitive Newtonian level (see Section II.3.2. In fact, the exponential character of the isounit emerged when turning nonconservative non-Hamiltonian Newtonian systems into an identical isotopic form, and this feature persists under operator formulations.

Alternatively, on mathematical grounds, one can see the emergence of the exponential structure of the isounit from the isodifferential of Section I.3.4. In this case, the use of the expressions of the type $\hat{I} = e^{A(r)}$ implies the cancellation between $\hat{T}$ and $\hat{I}$ in the additive term, namely, $r \times T \times d\hat{I} = r \times \partial_r A dr$.

It is hoped the reader can see that, besides all the arguments presented in Papers I and II, the LP symmetry cannot possibly be exact for the scattering region under the sole admission of its nonspherical, inhomogeneous and anisotropic character, in favor of the exact universal LPS isosymmetry. The isotopies of quantum mechanics of paper II and of the scattering theory presented in this paper, then follow rather inevitably.

2. Elements of Isoscattering Theory.

2.1. Isoscattering Amplitude. By assuming a knowledge of the content and terminology of Papers I and II, we can now expeditiously proceed with the formulation of the desired isoscattering theory. For clarity, we shall write most of the formalism in its projection on conventional spaces so as to show the differences with conventional formulations.

An important difference between the quantum and isotopic scattering theory is the assumption in the following. The conventional scatering theory assumes the speed of light $in\ vacuum\ c$ and related LP symmetry as being unchanged in the $hyperdense\ medium$ inside the scattering region. By contrast, the isoscattering theory assumes that the speed of light in the interior problem is the local avriable $C = c/n_4 = c \times b_4$ with universal LPS isosymmetry. However, when the scattering problem is studied on the Minkowski-Santilli isospacetime over the isoreals, the speed of light in the interior problems remains indeed that in vacuum, as recalled in Section II-2. These views imply that the characterization of the density of the scattering region indicated earlier is given by the index of refraction $n_4 = 1/b_4$, a notion absent in quantum theories.

The isoscattering theory alsop assumes that the geometry in the interior of the scattering region is mutated by its medium. This mutation is represented with the transition from the Euclidean space $E(r, \delta, \mathcal{R}$ over the reals $\mathcal{R}$ characterizing the vacuum (intended as empty space), to the Euclid-Santilli isospace $\hat{E}(\hat{r}, \hat{\delta}, \hat{\mathcal{R}}$ over the isoreals $\hat{\mathcal{R}}$ characterizing the hyperdense medium inside the scattering region.

Additionally, we assume the conservation of the volume of the scattering region in the transition from the conventional to the isotopic treat-

ment that can be expressed by the condition

$$Det\ \hat{\delta} = (b_1^2 \times b_2^2 \times b_3^2) = (b_1 \times b_2 \times b_3)^2 = 1 \tag{2.1}$$

However, it should be stressed that this condition can be easily relaxed later on, e.g., in the transition from low to high energy scattering in which the scattering volume definitely is not preserved. We finally assume for simplicity that the characteristic b-quantities have been averaged to costants because the scattering region is inspected from the outside.

The above assumptions require the replacement of the conventional spherical coordinates with the isospherical form (3.36), including the lifting of conventional angles into the isoangles

$$\hat{\theta} = \theta \times \hat{I}_\theta,\ \ \hat{\phi} = \phi \times \hat{I}_\phi,\ \hat{I}_\theta = b_3,\ \hat{I}_\phi = b_1 \times b_2, \tag{2.2}$$

In the simple case here considered, we then have the following trivial identities between isodifferentials and differentials

$$\hat{d}\hat{\theta} = b_3^{-1} \times d(\theta \times b_3) = d\theta,\ \ \hat{d}\hat{\psi} = (b_1 \times b_2)^{-1} \times d(\phi \times b_1 \times b_2) = d\phi \tag{2.3}$$

But we have the expression

$$\hat{d}\hat{cos}\hat{\phi} = \hat{sin}\hat{\phi} \times \hat{d}\hat{\phi} = (b_1 \times b_2)^{-1} \times sin(\phi \times b_1 \times b_2) \times d\phi, \tag{2.4}$$

that illustrates nontrivial departures between conventional and isotopic treatments despite the simplest possible assumptions made above.

The $solid\ isoangle$ is evidently given by

$$\hat{d}\hat{\Omega} = \hat{d}\hat{\theta} \times \hat{d}\hat{cos}(\hat{\phi}) =$$

$$= (b_1 \times b_2)^{-1} d\theta \times d\phi \times sin(\phi \times b_1 \times b_2), \tag{2.5}$$

with isointegral

$$\hat{\Omega} = \Omega \times \hat{I}_\Omega = \Omega \times (b_1 \times b_1 \times b_3)^{-1} = \hat{\int}\hat{\int} \hat{d}\hat{\theta} \times \hat{d}\hat{\phi} \times \hat{sin}\hat{\phi} =$$

$$= b_3^{-1} \times \int \int d\theta \times d\phi \times sin(\phi \times b_1 \times b_2), \tag{52.6}$$

and final expression

$$\Omega = (b_1 \times b_2) \times \int \int d\theta \times d\phi \times sin(\phi \times b_1 \times b_2), \tag{52.7}$$

that also illustrates the differences between conventional and isotopic treatments despite the preservation of the scattering volume (but not necessarily of the surface).

The *isoscattering amplitude* $\hat{f}(\hat{\theta}, \hat{\psi})$ can be defined in its most elementary form via the expression

$$e^{i \times k \times z} + \frac{\hat{f}(\hat{\theta}, \hat{\phi})}{\hat{r}} \hat{\times} e^{i \times k \times r}, \tag{2.8}$$

where $e^{i \times k \times z}$ and $e^{i \times k \times r}$ are the *conventional* incoming and scattered waves because we have assumed the exact validity of quantum scattering theories outside the scattering region.

The projection of Eq. (2.8) on a conventional Euclidean space can be written

$$e^{i \times k \times z} + \frac{\hat{f}(\theta \times b_3, \phi \times b_1 \times b_2)}{r} \times e^{i \times k \times r}, \tag{2.9}$$

where we assume the reader is aware that the isodivision in Eq. (2.8) allowing the replacement of $\hat{r}$ with r.

The *isodifferential cross section* is then given by

$$\hat{d}\hat{\sigma} - = |\hat{f}(\hat{\theta}, \hat{\phi})|^2 \hat{\times} \hat{d}\hat{\Omega}, \tag{2.10}$$

and the *total cross section* assumes the form

$$\sigma = \hat{\int} \hat{d}\hat{\sigma} = \hat{\int} |\hat{f}(\hat{\theta}, \hat{\phi})|^2 \hat{\times} \hat{d}\hat{\Omega} = \int |\hat{f}(\hat{\theta}, \hat{\phi})|^{\times} d\Omega. \tag{2.11}$$

As recalled in Section I.1, the cross section is a number that, as such, is independent from the selected elaboration. The novelty of the isoscattering theory is the mutation of the scattering amplitude, whose implications will be elaborated below.

2.2. Isoscattering Matrix. According to our assumptions of Section I.2, the initial and final states, $|i>$, $|f>$, respectively, are defined on

a conventional Hilbert space $\mathcal{H}$ over a conventional quantum field $\mathcal{C}$, as denoted by the lack of "hat" in these states. However, by central assumption, their interconnection is done via isomechanics on the Hilbert-Santilli isospace $\hat{\mathcal{H}}$ over the isofield of complex isonumbers $\hat{\mathcal{C}}$ with isoinner product (3.15) of Paper I and isounit (1.4). Therefore, the *isoscattering matrix* is defined by (see Ref. [6b], Chapter 12, for a review of earlier works and references)

$$< i|\hat{\times}\hat{S}\hat{\times}|f > \;\; = \;\; < i| \times T \times \hat{S} \times T \times |f >, \tag{2.12}$$

with basic isounitarity property

$$< i| \times T \times (\hat{S} \times T \times |f >) \; = \; (< i| \times T \times \hat{S}^{\dagger}) \times T \times |f > \tag{2.13}$$

namely

$$\hat{S}\hat{\times}\hat{S}^{\dagger} = \hat{S}\hat{\times}\hat{S} = \hat{I}, \tag{2.14}$$

or

$$\Sigma_f \hat{S}_{fi} \times T_{ik} \times \hat{S}_{fk} = \hat{\delta}_{ik} = \hat{I} \times \delta_{ik}, \tag{52.15}$$

expressing the conservation of probability on isospace over isofields.

It should be indicated that *the isoscattering matrix is an isomatrix,* namely, its elements are isonumbers $\hat{n} = n \times \hat{I}$. Consequently, the isotopic element in the isoinner product (5.11) can be eliminated with the reduction of type (4.35a)

$$\hat{S} = \tilde{S} \times \hat{I} \tag{2.16}$$

under which we can regain the conventional form

$$< i| \times T \times \hat{S} \times T \times |f > \;\; = \;\; < i| \times \tilde{S} \times |f > . \tag{2.17}$$

However, the nontrivial character of the isoscattering theory is that the reduced matrix $\tilde{S}$ is *nonunitary,*

$$\tilde{S} \times \tilde{S}^{\dagger} \neq I. \tag{2.18}$$

as it is the case also for Eq. (5.14). The latter property is crucial to guarantee the exiting put of the class of unitary equivalence of quantum scattering theories and, in its absence, novelty would only be illusory. Note

that *without the isotopic formulation, nonunitary scattering theories would be inconsistent* [6=11].

The *isotransition probability* that states $|i>$ are turned into the states $|f>$ is then given by

$$\hat{P}_{fi} = \hat{S}^{\dagger}_{fi} \times T \times \hat{S}_{fi} = (\tilde{S}^{\dagger}_{fi} \times \tilde{S}_{fi}) \times \hat{I}, \tag{52.19}$$

with evident *total isoprobability*

$$\hat{P}_{tot} = \Sigma_f \hat{S}^{\dagger}_{fi} \times T \times \hat{S}_{fi}. \tag{2.20}$$

2.3. Isopropagators. We now construct the isotopic image of quantum propagators, here called *isopropagators,* and then compare the resulting isoseries with a conventional expansion. We maintain the conventional assumption of the Hamiltonian being composed by two parts,

$$H = H_0 + H_1, \tag{2.21}$$

and consider the Schrödinger-Santilli isoequation (4.14) under the simplified assumption that the isounit of time is 1, namely, that time is not lifted, $\hat{t} = t$,

$$i \times \partial_t \hat{\psi}(t, r) = [H_0(r, p) + H_1(r, p)] \times T(t, r, p, \psi, \partial\psi, ...) \times \hat{\psi}(t.r). \tag{2.22}$$

To avoid the venturing of superficial technically unsubstantiated perceptions (to their peril), readers should be aware that the isoscattering theory allows the consistent inclusion, for the first time to our knowledge, of *nonlinear effects,* inclusion that would imply basic inconsistencies for conventional scattering theories indicated in reference to Eq. (4.15).

Additionally, the isoscattering theory allows the verification of causality via an irreversible treatment of irreversible scattering processes via the use of the broader genomathematics [15,16], a condition also impossible for quantum scattering theories, with the understanding that the proper treatment of irreversibility requires the covering Lie-admissible genoscattering theory.

The isopropagator $\hat{G}(t,r;t'r')$ is then defined by

$$i\times\partial_t\hat{G}(t,r;t',r')-(H_0+H_1)\times T\times\hat{G}(t,r;t',r')=\hat{\delta}(t'-t)\times\hat{\delta}^3(r'-r), \quad (2.23)$$

where $\hat{\delta}^3$ and $\hat{\delta}$ denotes the Dirac-Myung-Santilli isodelta function of hadronic mechanics (Section 3.5).

Recall that quantities with an isoscalar structure show the factorized isounit when projected on conventional spaces, such as $\hat{g}=g\times\hat{I}$. In this case we have the simplification of the isoproduct of an isofunction by another quantity $\hat{g}\hat{\times}A=g\times\hat{I}\times T\times A=g\times A$.

However, the isodelta is an *isodistribution* and, as such, it does not admit the factorization of the isounit, as it is the case for the isodifferential. Since the isodelta has no factorization of the isounit, the same holds for the isopropagator $\hat{G}(t,r;t',r')$.

Assuming that $\hat{G}_0(t,r;t',r')$ is the isopropagator for H_0, we then have

$$\hat{G}(t,r;t',r')=\hat{G}_0(t,r;t',r')+$$

$$+\int\int dt'\times d^3r'\times\hat{G}(t,r;t_1,r_1)\times T\times H_1(t,r)\times T\times\hat{G}_0(t,r;t',r'). \quad (2.24)$$

2.4. Convergent Isoexpansions. At this point, we subject Eq. (5.24) to a power isoexpansion in terms of H_1 as in the original case [1], however, without the conventional restriction that the interacting term H_1 is small. By recalling that the formulation is on $\hat{\mathcal{H}}$, we have the expression

$$\hat{G}(t,r;t',r')=\hat{G}_0(t,r;t',r')+$$

$$+\int\int dt'\times d^3r'\times\hat{G}_0(t,r;t',r')\times T\times H_1(t_1,r_1)\times T\times\hat{G}_0(t,r;t_1r_{,1})+$$

$$+\int\int dt'\times d^3r'\times\hat{G}_0(t,r;t_1,r_1)\times T\times H_1(t_1,r_1)\times T\times\hat{G}_0(t_1,r_1;t',r')+$$

$$+\int\int dt'\times d^3r'\times\hat{G}_0(t,r;t_1,r_1)\times T\times H_1(t_1,r_1)\times$$

$$\times T\times\hat{G}_0(t_1,r_1;t_2,r_2)\times T\times H_1(t_2,r_2)\times\hat{G}_0(t_2,r_2;t',r')+... \quad (2.25)$$

Similarly, by assuming that $\hat{\psi}_0(t,r)$ is the isoeigenfunction of H_0,

$$i \times \partial_t \hat{\psi}_0(t, r = H_o \times T \times \hat{\psi}_0(t,r), \tag{2.26}$$

we have the isoexpansion for the wave isofunction

$$\hat{\psi}(t.r) = \hat{\psi}_0(t,r) +$$

$$+ \int\int dt_1 \times d^3 r_1 \times \hat{G}_0(t,r;t_1,r_1) \times T \times \hat{\psi}_0(t_1.r_1) + ... \tag{2.27}$$

We now assume for $t \to -\infty$

$$\psi_0(t,z) = \psi_i(t,r) e^{i \times k \times r} \tag{2.28}$$

Then we have the expression

$$\hat{\psi}(t.r) = \psi_0(t,r) +$$

$$+ \int\int dt_1 \times d^3 r_1 \times \hat{G}_0(t,r;t_1,r_1) \times T \times \psi_0(t_1.r_1) + ... \tag{2.29}$$

and we have the explicit form of the isoscattering matrix

$$\hat{S}_{fi} = \hat{I} + i \times \int dt' \times d^3 r \times \psi_0(tt', r')^\dagger \times T \times H_1(t', r') \times \psi_0(t', r + ... \tag{2.30}$$

It is evident that all the above expansions are strongly convergent, not only because the isotopic eleent verifies the condition

$$|T| \ll |H_1|. \tag{2.31}$$

but also because the isopropagator no longer admits the divergence of the conventional propagator for $t = t', r = r'$, thus confirming a main objective of this paper.

2.5. Prediction of Mini-Black-Holes. Let us consider the isogravitational content of the isoscattering theory outlined in Section 2.5 of Paper II. Recall the identification of the conventional Riemannian metric, such

as the Schwartzschild metric in the coordinates (θ, ϕ, r, t) , with the isometric $\hat{\eta}$ in the very structure of the isogamma matrices, Eqs. (2.35) of Paper II,

$$ds^2 = r^2(d\theta^2 + sin^2 d\theta^2 + d\phi^2) + (1 - \frac{2 \times M}{r})^{-1} \times dr^2 -$$

$$-(1 - \frac{2 \times M}{r} \times dt^2 \equiv \hat{T}_{sch} \times \eta \equiv \hat{\eta}, \tag{2.32}$$

which is represented with *gravitational isounit and isotopic element*

$$\hat{T}_{sch} = Diag.[1, 1, (1 - \frac{2 \times M}{r})^{-1}, (1 - \frac{2 \times M}{r})], \tag{2.33a}$$

$$\hat{I}_{sch} = Diag.[1, 1, (1 - \frac{2 \times M}{r}), (1 - \frac{2 \times M}{r})^{-1}]. \tag{2.33b}$$

We also recall that the conventional Dirac equations represents an *electron in vacuum,* as well known, while the isotopically lifted equation represents *the same electron when immersed within a hyperdense hadronic medium.*

It is then easy to see that *the isoscattering theory predicts the possible creation of mini black holes under sufficiently high energies,* as evident from the singularities of isometric (2.32) that now constitutes the actual metric of the scattering region.

Needless to say, there is no possibility to predict at this stage of the knowledge the threshold of *energy density* needed to trigger a mini-black-hole, and this explains the importance of lifting the scattering theory into a form admitting of a quantitative representation of the energy density of the scattering region.

Recall that, for the conventional scattering theory, the metric for the scattering region is the conventional Minkowski metric, thus without any predictive capacity for mini-black-holes. Consequently, the above prediction of the isoscattering theory and related isoline element (2.32) is sufficient to illustrate the need for serious caution and scrutiny before embarking in extremely high energy scattering experiments based on insufficient quantum methods that can at best produce controversial results.

3. Isotopies of Feynman's Diagrams without Divergences

3.1 Conventional Feynman Diagrams We are now equipped to tackle a central aspect of our research, the isotopies of Feynman's diagrams, first studied by Animalu [7] and here referred to as *iso-Feynman diagrams.* These isotopies require a re-inspection in this paper because originally conducted without the use of isomathematics, thus having the shortcomings of Refs. [6-12] of Paper I.

We begin by recalling the main features of conventional Feynman diagrams. then construct their isotopic images, and outlining the rules for computing isoscattering cross-sections from iso-Feynman diagrams.

As is well-known (see, e.g., Ref. [8]), Feynman's diagrams (also called path-integral technique in QED) comprise a representation of fundamental elementary particles (e.g., electrons e^-), their antiparticles (e.g. positrons e^+), and their annihilation into two photons, e.g.,

$$e^- + e^+ \to \gamma + \gamma, \tag{3.1}$$

in which: the electron e^- is represented by a "point" moving forward in time and the positron e^+ by another "point" moving backwards in time; the pair annihilates at the intersection of the lines joining the two point-particle trajectories as shown in Figure 1(a)); the emitted photons are represented by wiggly lines radiating away from the point of annihilation, and Figure 1(b) represents Coulomb interactions via virtual photon exchange.

The scattering cross-sections is then calculated by following a number of *Feynman rules* designed to reproduce, at the lowest order, classical results found with conventional quantum mechanics. The full power of the method is realized, however, in calculating the radiative corrections and higher order terms and, as we shall see in this section, corrections from non-Hamiltonian forces activated in the hadronic scattering region.

3.2. The Problem of Antiparticles in Feynman's Diagrams. As indicated in paper I, Section 2, in these initial studies we shall continue to use the 20th century formalism for antiparticles. Nevertheless, the reader should be aware of the novel *isodual theory of antimatter* that has achieved

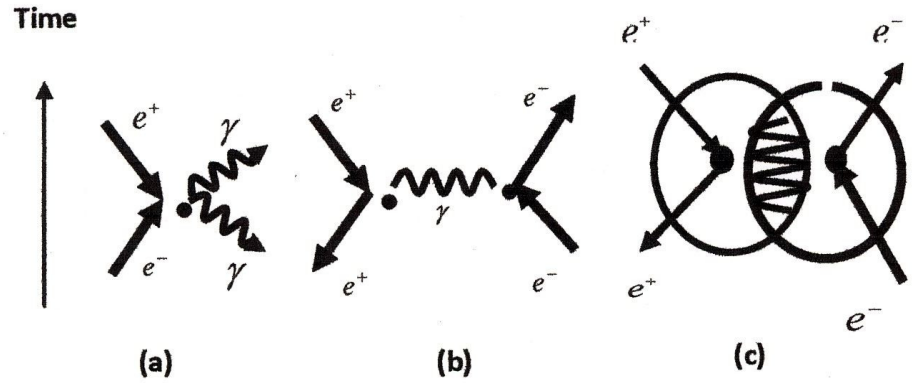


Figure 1: *Conventional Feynman's graphs for: (a)* $e^-e^+ \to \gamma + \gamma$ *annihilation into two photons; (b)* $e^- - e^+$ *Coulomb interaction via "virtual" photon exchange; and (c) nonunitary representation of non-Hamiltonian interactions of extended wave packets due to overlapping in the annihilation process (3.1) or the' model of the fusion process* $e^+ + e^- \to \pi^0$

the first known axiomatically consistent representation of antimatter at all levels, from Newtonian mechanics to second quantization, while being equivalent to charge conjugation in its operator form.

The new theory of antimatter has been motivated by clear insufficiencies of the classical theories of the 20th century, such as the absence of any differentiation between *neutral* matter and antimatter, problematic aspects in the classical treatment of *charged* antiparticles (because their operator image is a conventional "particle," rather than the needed charge conjugated "antiparticle"), and other problematic aspects requiring a resolution on serious scientific grounds.

The map from matter to antimatter at all levels is performed by an operation called *isoduality,* represented with the upper symbol d, essentially given by anti-Hermiticity, that must be applied, for consistency, to all possible quantities and their operations. Consequently, a central feature of the isodual theory is that *all physical quantities of antimatter are measured with negative units,* thus including negative units of time, en-

ergy, linear momentum, etc.

The above formulation of antimatter resolves the known uneasiness in Feynman's use of motions backward in time, because a motion backward in time referred to a negative unit of time is as causal as motion forward in time referred to a positive unit of time.

Additionally, this feature renders fully causal the negative energies of Dirac's equation when, again, referred to a negative unit, and provides a new interpretation of Dirac's equation as directly representing an electron-positron pair at the level of first quantization without any need for the "hole theory" or second quantization, because the isodual theory of antimatter holds at the *classical* level, let alone for first quantization [9].

Additionally, these studies have identified a new fundamental symmetry of spacetime, called *isoselfduality* (essentially the invariance under anti-Hermiticity as verified by the imaginary unit $i \equiv i^d$), that essentially providing a deeper representation of the invariance of a particle-antiparticle pair under charge conjugation. This new symmetry is verified by the l.h.s. of Eq. (3.1), since $(e^- + e^+)^d = e^+ + e^-$, but it is violated in the r.h.s. because $\gamma^d \neq \gamma$.

This new spacetime symmetry (that could be verified by our entire universe in the event made up of matter and antimatter in equal amounts) suggests the verification of the same symmetry in the r.h.s. of Eq. (3.1). This has been achieved in Ref. [9] via the differentiation between the *conventional photon* γ emitted by matter and the *isodual photon* γ^d emitted by antimatter. The latter is predicted as being physically different than that emitted by matter, e.g., because repelled by the gravitational field of matter and having other experimentally verifiable features that may initiate, in due time, the new field of *experimental antimatter astrophysics.*

As a result,. the new isodual theory of antimatter replaces Eq. (3.1) with in the following isoselfdual invariant reaction [9]

$$e^- + e^{-d} \ \rightarrow \ \gamma + \gamma^d, \tag{3.2}$$

where $e^d \equiv e^+$ since isoduality is equivalent to charge conjugation.

It is evident that the above occurrence requires, alone, a re-inspection of the entire formulation of the Feynman's diagrams that we cannot possibly achieve in these first papers to prevent a prohibitive length and that

has to be deferred to a subsequent study.

3.3 Isotopies of Feynman Diagrams. Figure 1(c) represents the proposed isotopic image of non-Hamiltonian interactions of extended wave packets due to overlapping, as predicted by hadronic mechanics since its inception, e.g., for model of theπ^o synthesis from electrons and positrons, the synthesis of neutrons from protons and electrons, or the synthesis of hadrons at large from lighter particles (see the excellent review by Kadeisvili [10]).

In terms of the isounit $\hat{I}$ and isotopic element $\hat{T}$, here indicated as $\hat{I}_{st}$ and T_{st} for the carrier space of a relativistic hadronic mechanics, the correspondence between free-particle Feynman propagators in conventional relativistic theory and their isotopic image in hadronic mechanics, can be characterized as follows:

$$[S_F(x) = (\gamma^\mu p_\mu + im)\Delta_F(x)] \longrightarrow [\hat{S}_F(\hat{x}) = (\hat{\eta}^{\mu\nu}_{st} \times \hat{\gamma}_\mu \times \hat{p}_\nu + i \times \hat{m}) \times T_{st} \times \hat{\Delta}_{\hat{F}}(\hat{x})], \tag{3.3a}$$

$$[\Delta_F(x) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-px}}{p^2 - m^2 + i\epsilon}] \longrightarrow [\hat{\Delta}_F(\hat{x}) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-p \times T_{st} \times x}}{\hat{p}^{\hat{2}} - \hat{m}^{\hat{2}} + i \times \hat{\epsilon}}] \tag{3.3b}$$

with corresponding expression in momentum 4-vector space:

$$[S_F(p) = (\gamma^\mu p_\mu + m)\Delta_F(p^2) = \frac{\gamma^\mu p_\mu + m}{p^2 - m^2 + i\epsilon}] \longrightarrow$$

$$[\hat{S}_F(\hat{p}) = (\hat{\eta}^{\mu\nu}_{st} \times \hat{\gamma}_\mu \times \hat{p}_\nu + i \times \hat{m}) \times T_{st} \times \hat{\Delta}_F(\hat{p}^2) = \frac{(\hat{\eta}^{\mu\nu}_{st} \times \hat{\gamma}_\mu \times \hat{p}_\nu + i \times \hat{m}) \times T_{st}}{\hat{p}^{\hat{2}} - \hat{m}^{\hat{2}} + i \times \hat{\epsilon}}] \tag{3.4}$$

3.4. Isotopic Inclusion of Electromagnetic interactions. In the presence of an external electromagnetic field, the solution of the (regular) Dirac-Santilli isoequation takes the form

$$\hat{\Psi} = \hat{\psi}(\hat{x}) + \hat{e}\hat{\times}\hat{\int} \hat{d^4}\hat{x'}\hat{\times}\hat{S}_f(\hat{x} - \hat{x'})\hat{\times}\hat{\gamma}\hat{.}\hat{A}(\hat{x'})\hat{\times}\hat{\Psi}(\hat{x'})$$

$$= \hat{\psi}(\hat{x}) + \hat{e}\hat{\times}\hat{\int} \hat{d^4}\hat{x'}\hat{\times}\hat{S}_f(\hat{x} - \hat{x'})\hat{\times}\hat{\gamma}\hat{.}\hat{A}(\hat{x'})\hat{\times}\hat{\psi}(\hat{x'})$$

$$+\hat{e}^{\hat{2}}\hat{\times}\hat{\int}\hat{d}^4\hat{x}'\hat{\int}\hat{d}^4\hat{x}''\hat{\times}\hat{S}_f(\hat{x}-\hat{x}')\hat{\times}\hat{\gamma}\hat{.}\hat{A}(\hat{x}')\hat{\times}\hat{\psi}(\hat{x}')\hat{\times}\hat{S}_f(\hat{x}'-\hat{x}'')\hat{\times}\hat{\gamma}\hat{.}\hat{A}(\hat{x}'')\hat{\times}\hat{\psi}(\hat{x}'')+... \tag{3.5}$$

This leads to a formal definition of the Feynman-Animalu isopropagator either as a series:

$$\hat{S}'_f(\hat{x},\hat{x}') = \hat{S}_f(\hat{x}-\hat{x}') + \hat{e}\hat{\times}\hat{\int}\hat{d}^4\hat{x}''\hat{\times}\hat{S}_f(\hat{x}-\hat{x}'')\hat{\times}\hat{\gamma}\hat{.}\hat{A}(\hat{x}'')\hat{\times}\hat{S}_f(\hat{x}'-\hat{x}'')+... \tag{3.6}$$

or as an integral equation:

$$\hat{S}'_f(\hat{x},\hat{x}') = \hat{S}_f(\hat{x}-\hat{x}')+\hat{e}\hat{\times}\hat{\int}\hat{d}^4\hat{x}''\hat{\times}\hat{S}_f(\hat{x}-\hat{x}'')\hat{\times}\hat{\gamma}\hat{.}\hat{A}(\hat{x}'')\hat{\times}\hat{S}'_f(\hat{x}',\hat{x}''). \tag{3.7}$$

where $\hat{\gamma}\hat{.}\hat{A}(\hat{x}') \equiv \hat{\eta}^{\mu\nu}_{\hat{s}\hat{t}} \times \hat{\gamma}_\mu \times \hat{A}_\nu(\hat{x}')$ and $\hat{A}_\nu(\hat{x}')$ is the iso-electromagnetic field 4-vector potential given by the corresponding iso-gauge principle[xx].

To elaborate the basic physical concepts in terms of Feynman diagrams for electron scattering with an electromagnetic field in interior dynamical conditions, we show in Figure 4 the two characteristic differences in 1st and 2nd quantization schemes. The iso-scattering matrix is given in first quantization scheme by

$$\hat{S}_{f,i} = lim_{t\to\infty}\hat{\int}\hat{d}^3\hat{x}\hat{\times}\hat{\psi}^{+\hat{s}'}_{\hat{p}'}\hat{\times}\Psi^{\hat{s}}_{\hat{p}} \tag{3.8}$$

and $\Psi^{\hat{s}}_{\hat{p}}$ is the exact solution given as in Eq.(6.5) by

$$\Psi^{\hat{s}}_{\hat{p}}(\hat{x}) = \hat{\psi}^{\hat{s}}_{\hat{p}}(\hat{x}) + \hat{e}\hat{\times}\hat{\int}\hat{d}^4\hat{x}'\hat{\times}\hat{S}_f(\hat{x}-\hat{x}')\hat{\times}\hat{\gamma}\hat{.}\hat{A}(\hat{x}')\hat{\times}\hat{\Psi}^{\hat{s}}_{\hat{p}}(\hat{x}') \tag{3.9}$$

with the iso-normalization

$$\hat{\int}\hat{d}^3\hat{x}\hat{\times}\hat{\psi}^{+\hat{s}}_{\hat{p}}(\hat{x})\hat{\times}\hat{\psi}^{\hat{s}'}_{\hat{p}'}(\hat{x}) = \hat{\delta}_{\hat{s}\hat{s}'}\hat{\times}\hat{\delta}^3(\hat{\mathbf{p}}-\hat{\mathbf{p}}') \tag{3.10}$$

Note that the correspondence principle in 1st quantization scheme involves a lifting of the Coulomb vertex in QED into the approximate Yukawa vertex in hadronic mechanics, and additionally involves the lifting from Bose-Einstein to Fermi-Dirac statistics in 2nd quantization scheme, i.e., mutation of spin under sufficiently high energies.

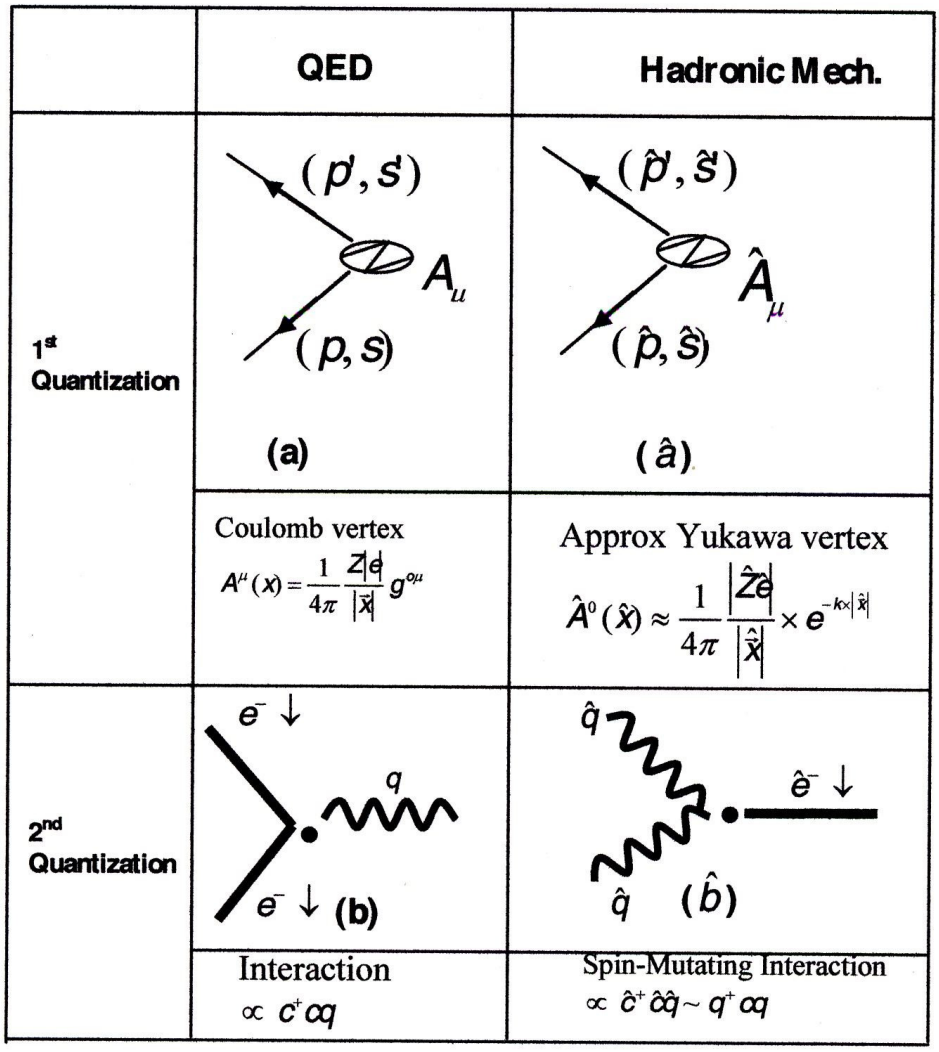


Figure 2: Correspondence between the Feynman graphs for fermion-fermion interaction vertex with boson emission in 1st and 2nd quantization schemes in QED and the isotopic images for the boson-boson interaction vertex with single fermion spin emission in HM.

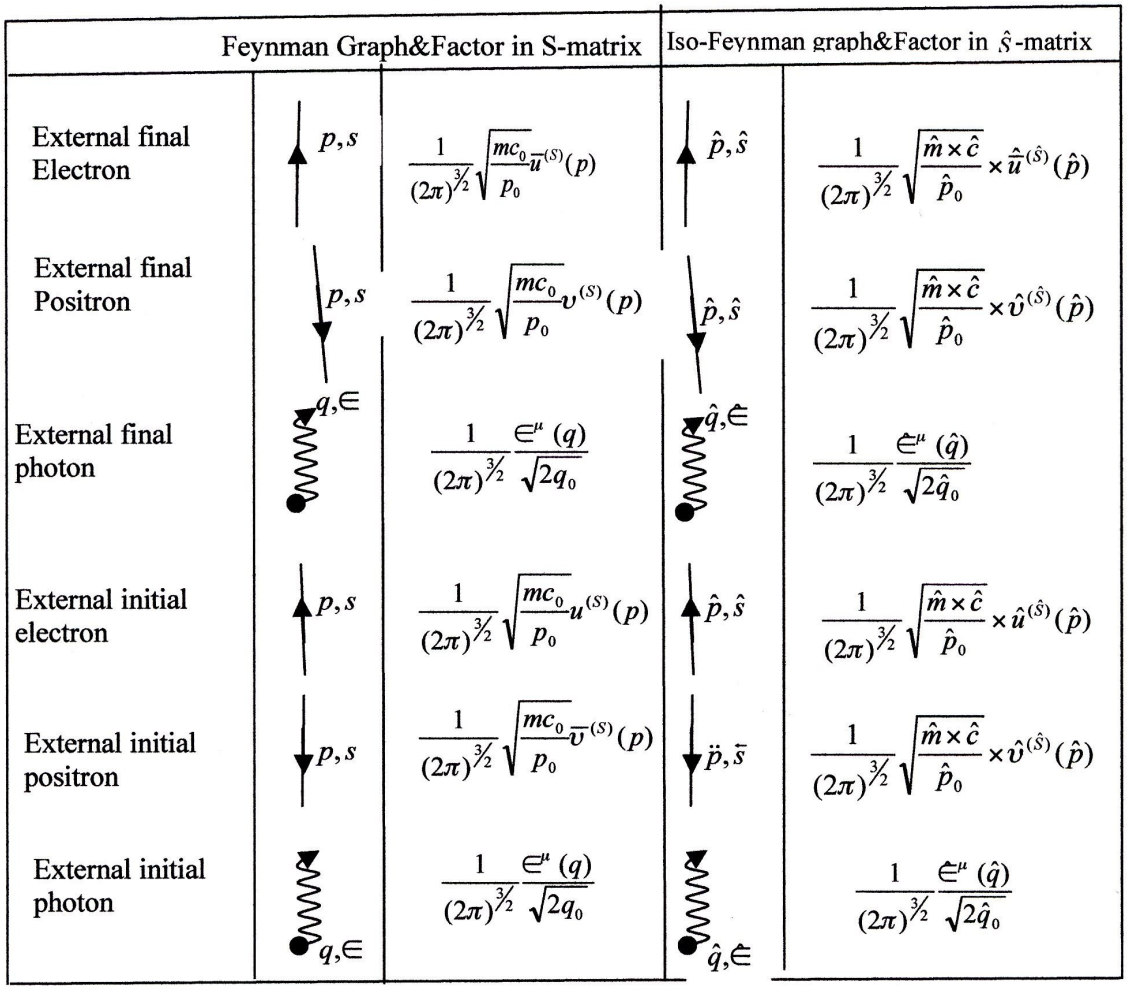


Figure 3: Feynman Graphs/Rules and their Isotopic Images

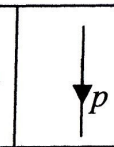
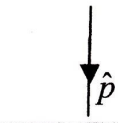
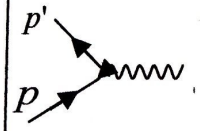
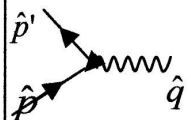
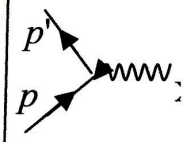
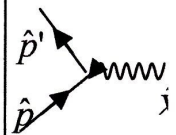
Feynman Graph & factor in S-matrix			Iso-Feynman graph & factor in $\hat{S}$-matrix	
Internal Electron		$iS_F(p)$, given by the expression in Eq, (6.2)		$i\hat{S}_F(\hat{p})$ given by the expression in Eq.(6.2)
Internal photon		$-ig^{\mu\nu}D_F(q^2)$ $=\frac{-ig^{\mu\nu}}{q^2+i\in}$		$-i\hat{g}^{\mu\nu}\hat{D}_F(\hat{q}^2)$ $=\frac{-i\hat{g}^{\mu\nu}}{\hat{q}^2++(m_\phi\hat{c})^2+i\hat{\in}}$
Vertex		$-i\sqrt{4\pi}e\gamma^\mu(2\pi)^4\times$ $\gamma^\mu\times\delta^{(4)}(p'-p-q)$		$-i\sqrt{4\pi}e\hat{\gamma}^\mu(2\pi)^4\times$ $\hat{\gamma}\times\hat{\delta}^{(4)}(\hat{p}'-\hat{p}-\hat{q})$
Coulomb Vertex		$\frac{-i4\pi Z\lvert e\rvert e}{\lvert\vec{p}'-\vec{p}\rvert^2}\gamma^0\times$ $(2\pi)\delta(p_0'-p_0)$		$\frac{-i4\pi Z\lvert\hat{e}\rvert\hat{e}}{\lvert\hat{\vec{p}}'-\hat{\vec{p}}\rvert^2+(m_\phi\hat{c})^2}\times\hat{\gamma}^0\times$ $(2\pi)\hat{\delta}(\hat{p}_0'-\hat{p}_0)$

Figure 4: Continuation of Figure 5

The correspondence between Feynman graphs/rules and their isotopic images for computation of contributions to the S-matrix in QED of spin- particles are summarized in the Figures 5, 6, as well as the rules:

(1) $\hat{\int}\frac{\hat{d}^4p}{2\pi^4}$ for each internal line.

(2) Overall sign $(-)^{L+P}$ where L is the number of closed electron loops and P is the permutation of the external particles.

(3) Phase space of final particle involves $d^3p...$; and

(4) Flux of particles is $V\frac{\mathbf{V_1}-\mathbf{V_2}}{(2\pi)^6}$.

3.5. Concluding remarks. Lagrange, Hamilton, Jacobi and other founders of mechanics stated that nature cannot be entirely reduced to potential interactions solely representable with quantities we call today Lagrangians and Hamiltonians, for which reason they wrote their celebrated analytic equations with $external\ terms$ representing interactions not admitting a potential energy (for historical references and comments, see Refs. [11]).

Due to the successes of purely Lagrangian and Hamiltonian theories, such as special relativity and quantum mechanics, the external terms were removed from the analytic equation in the mainstream physics of the 20th century, thus abstracting all events in the universe to potential interactions among point-like particles moving in vacuum, an abstraction that was also implemented for the scattering theories.

However, the $No\ Reduction\ Theorem$ of paper I, Section 2, has confirmed the historical legacy of Lagrange, Hamilton and Jacobi, by establishing in particular that the nonlinear, nonlocal and nonpotential interactions represented with external terms necessarily originate at the ultimate level of nature, that of deep mutual penetration and overlapping of particles, thus requiring a broadening of the scattering theory for their inclusion.

At any rate, the rigorous implementation of the axioms of quantum mechanics without $ad\ hoc$ manipulations, and their inherent point-like abstraction of particles, essentially reduce most scattering process among charged particles to Coulomb interactions. This causes uneasiness in the conventional explanation of multi-particle productions via second quantization, since the latter events are clearly visible in detectors, thus ex-

pected as being interpreted at the *semiclassical* level, let alone that of *first* quantization.

Mutatis mutandae, the admission of any particle dimension of the same order of magnitude of the scattering region, causes their deep mutual overlapping, with consequential need to include in scattering processes precisely the nonlinear, nonlocal and nonpotential interactions originated by Lagrange,. Hamilton and Jacobi, since these contacts effects are unavoidable for the dynamics of extended particles.

Predictably, the inclusion of external terms in the analytic equation has dramatic implications since it causes the loss of most conventional mathematical and physical knowledge. In fact, the brackets of the time evolution of Hamilton's equation with external terms violate the conditions to characterize an algebra, let alone causes the loss of all Lie algebras (see Refs. [6] for their analysis at the foundation of hadronic mechanics).

This occurrence mandated very laborious efforts lasted for decades to construct basically new mathematics and mechanics capable of incorporating Lagrange's and Hamilton's external terms. The sole representation of the historical external terms achieving invariance over time was their classical representation via Santilli's isounit and related isotopic element that can be presented in these concluding remarks in their original as well as ultimate meaning. This scenario explains the delay of decades prior to being in a position of addressing the scattering problem.

The reasons for a nonunitary-isounitary broadening of conventional unitary scattering theory have been indicated throughout these papers and need not be repeated here. The possible significance of the former theory over the latter can solely be established in subsequent papers over a significant period of time.

Hence, we can conclude these remarks by bringing to the attention of the curious reader that the ultimate origin of the new isoscattering theory rests indeed the historical legacy of Lagrange and Hamilton on the external terms of their celebrated analytic equations, which terms, following their popular suppression in the 20th century, have re-emerged in all their historical, mathematical and physical relevance.

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